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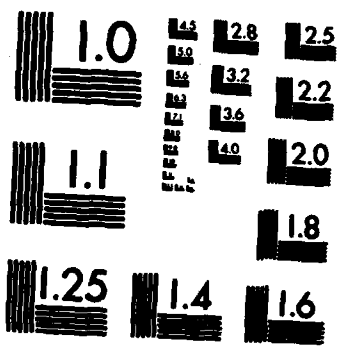

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By T.A. Shugar
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NCEL
Technical Note

A Numerical Study of Automated Dynamic Relaxation for Nonlinear Static Tensioned Structures

ABSTRACT The problem addressed in this research addresses ~~is~~ the lack of a robust structural analysis computational procedure for determining the initial equilibrium configuration and prestress state of tensioned structures. Three nonlinear iterative solution algorithms were considered: ~~in this study:~~ (1) the Newton algorithm; (2) the modified Newton algorithm; and (3) the automated dynamic relaxation (ADR) algorithm.

A large set of small cable test problems was designed to evaluate the robustness of each algorithm. In these test problems, the starting conditions were designed purposely to be onerous to test the ability of the algorithms to seek the correct static equilibrium position from rather arbitrary starting configurations.

The ADR algorithm proved to be more robust than the Newton algorithm, and the modified Newton algorithm was entirely unsatisfactory for this class of problems. Keywords: →

NAVAL CIVIL ENGINEERING LABORATORY PORT HUENEME CALIFORNIA 93043

METRIC CONVERSION FACTORS

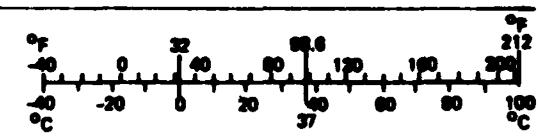
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The problem addressed in this research is the lack of a robust structural analysis computational procedure for determining the initial equilibrium configuration and prestress state of tensioned structures. Three nonlinear iterative solution algorithms were considered in this study: (1) the Newton algorithm, (2) the modified Newton algorithm, and (3) the automated dynamic relaxation (ADR) algorithm.

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20. Continued

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CONTENTS

INTRODUCTION

The problem addressed in this research is the lack of a robust structural analysis computational procedure for determining the initial equilibrium configuration and prestress state of tensioned structures. The solution of this problem is widely regarded as a major stumbling block to efficient design and analysis of tensioned structures, i.e., ocean cable structures and land-based fabric and cable structures.

The following are quotes from some investigators who are concerned with computational cable dynamics that relate to the difficulty of computing an initial equilibrium configuration solution:

1. Webster (1977), "Perhaps the most frustrating problem encountered in analyzing cable structures using displacement components is that of getting a stable initial configuration."

2. Liu (1977), "The results of dynamic simulation of the behavior of undersea cable structures depends strongly on the initial configuration of the system..."

3. Shields and Zueck (1984), "Techniques for simulating response of small deep water platforms using finite element modeling of the mooring legs are not presently available."

4. Webster (1984), "Our major numerical problems appear to be ill-conditioning (of the stiffness matrix) and extremely sensitive position dependent behavior in mooring systems. All of this is mainly in the static solutions."

Three iterative solution algorithms for nonlinear static problems were addressed in this study. The first two have been used in nonlinear finite element analysis of cable systems for many years and may be regarded as status quo solution algorithms. The third algorithm is a promising algorithm for structures exhibiting strong nonlinear behavior known as the automated dynamic relaxation (ADR) algorithm.

The ADR algorithm possesses some attractive theoretical features. These features provide for constant monitoring and control of the stability of the solution process by correcting and improving numerical conditioning of the model automatically. The status quo methods do not monitor or control numerical conditioning of the model that may exhibit pathological behavior particularly when an otherwise tensioned structure approaches a slack condition state.

A large set of small cable test problems was designed to evaluate the robustness of each algorithm. In these test problems, the starting conditions were designed purposely to be onerous to test the ability of the algorithms to seek the correct static equilibrium position from rather arbitrary starting configurations.

Objective

The objective of this research is to compare the robustness of full Newton and modified Newton solution algorithms with the ADR solution algorithm for highly kinematically, nonlinear, static, cable systems. The Newton-based algorithms represent the status quo solution methods while the ADR is a promising method for the class of problems considered.

Background

Leonard (1987) suggests that the task of structural analysis of cable systems and tensioned fabric systems can conveniently be viewed in terms of a Phase I and a Phase II problem. In the first phase the solution to the static, prestress configuration is sought. This phase is also referred to as form finding, shape finding, or the initial equilibrium problem. In the second phase the response due to the in-service loads is sought. The two-phase approach emphasizes that there must first be determined a static equilibrium configuration, about which either static or dynamic deflections will occur due to either prescribed static or dynamic working loads.

Both phases are characterized, in general, by nonlinear kinematic behavior. The first phase will most certainly be nonlinear, whereas the second phase may less frequently be found to be nonlinear. However, in general, solution schemes must be developed that anticipate nonlinear kinematic behavior for both phases. The solution schemes for each phase may be somewhat different because the first phase is always a static problem whereas the second phase may often be a dynamic problem. In the dynamic case, it has long been known that the inertia of the structural system does facilitate stability of the numerical solution.

The system of equations, which is the focus of the solution methods presented, is formed by a displacement-based finite element spatial discretization of an arbitrarily complex cable network. Cables systems are flexible and their stiffness ranges from zero when in a slack condition to significant values when in a taut condition. Thus they are characterized by nonlinear kinematic behavior so that the stiffness of the system depends on the displacements of the system. Further, the degree of nonlinearity is highly variable. Indeed, with sufficient tautness they may be linear in some cases. But when they possess substantial sag or when they approach a slack condition, they may become highly nonlinear. Since this entire range of nonlinearity can be experienced in an engineering application, the solution algorithm that will work best must possess accuracy and robustness.

Robustness of a solution algorithm relates to the degree to which the algorithm is foolproof in engineering application. It is principally this characteristic, along with accuracy, that is sought and investigated in the solution algorithms addressed in the present study. Other related performance characteristics such as convergence, speed, computational efficiency, and storage requirements are important, but of a secondary consideration. This is because it is believed that even the largest and most complex cable structures result in only moderately large systems of equations relative to typical finite element systems today.

(This may not be as true of tensioned fabric structural systems.) This is in part due to the small number of unknowns associated with cable elements as compared with continuum and structural elements. Even three-dimensional tensioned fabric structures are largely composed of the simplest of continuum elements. Further, in practical engineering applications, efficiency relates to the ability to get the project completed, as contrasted with the computational efficiency of the algorithm. Thus, the ability to converge to the correct solution in a reliable manner, even at the expense of speed and storage performance, is the key to a good solution algorithm for practical analysis of cable and tensioned fabric structures.

Unilateral tension stress behavior and allowance for slackness in cable systems introduces another form of nonlinearity that is present irrespective of large deflections and irrespective of any consideration for nonlinearity in external loads or in cable material response. One proper way to proceed is to supplement the linear or nonlinear restoring force equations of the cable system with constraint equations that determine whether or not a cable is slack. If the constraint equations are implemented as equalities, then the cable displacement variables are supplemented by Lagrange multiplier variables in the state vector of system unknowns. If the constraint equations are introduced as inequalities, then so-called slack variables* may also be introduced as unknowns. The practical computational impact of this is that the cable stiffness matrix necessary to properly describe and track slack response becomes indefinite and ill-conditioned, even for an otherwise linear cable system. Another, more simplified, way to proceed is to use truss elements and to remove the stiffness contribution of these elements to the structure stiffness matrix when they are in compression. Similarly, their stiffness contribution must be retrieved if and when the element deformation once again indicates extension.

*"Slack variables" is mathematical terminology from optimization theory. Interestingly, in the present application they would indeed represent a kinematic measure of slackness of a cable.

Any additional nonlinearities introduced into the system that are due to cable material nonlinearity, do not seem relatively to be that troublesome. Appropriate nonlinear constitutive models, once they are identified, would not add that much more computational complexity over and above that which is due already to appropriately account for large deflections, equations of constraint, and nonconservative excitation. That is, if appropriate nonlinear finite element solution algorithms can be found for the principal sources of nonlinearity in cable systems, they may be expected to be sufficiently general to work well also when material nonlinearity is present.

However, in this regard, mention should be made of the strength of cables, or more importantly their loss of strength under certain conditions. In the case of wire rope, Lucht and Donecker (1982) report that a kinked cable's strength may be reduced to as low as one-half the cable's rated strength. An interesting analysis of the dynamics to form kinks is presented by Yabuta, et al. (1982). Other strength reduction factors such as fatigue and corrosion must also be accounted for in conjunction with constitutive models used for predicting cable system performance.

Cable Roof Systems. Structural analysis of cable systems has been under development for many years, particularly for cable roof systems. Scalzi, et al., (1971) published an American Society of Civil Engineers (ASCE) Special Structures Committee report entitled "Cable Suspended Roof Construction, State-of-the-Art." A good deal of historical information is contained in this report. Part II of this report, "Analysis of Suspension Structures," is summarized briefly in the following for the purpose of bringing into relief the computer-based, finite element procedures that have been emphasized in practice since this report was written.

The ASCE report emphasized static analysis methods (with only brief mention of dynamic analysis). It explicates the difficulty of the Phase I problem that is typical of static, nonlinear cable systems in determining the initial shape, i.e., the equilibrium position under the initial loading consisting of dead load and prestress forces. It describes the necessity for extensive trial-and-error computations, directly referring to the kinematic nonlinearity of the Phase I problem. It even mentions the practice of constructing physical models to obtain reliable starting values for describing the suspension system shape to effectively initiate the trial-and-error computation. The main point that is made, regarding the initial shape, in the ASCE report and that also characterizes the nature of the Phase I problem, can be paraphrased in the following two sentences:

> The prescribed shape implies, by virtue of static equilibrium, a requisite prestress force. Alternatively, the prescribed prestress force implies, by virtue of static equilibrium, a requisite shape.

Consequently, in the design and analysis of cable systems (and tensioned fabric systems), one cannot simultaneously prescribe both the shape and the prestress force independently of equilibrium. One can prescribe system parameters such as unstrained cable length, cable stiffness, coordinates of support points, spans, and the dead load. Then by assuming some "guessed-at" cable configuration as a starting point and invoking equilibrium conditions, compute an initial shape and the corresponding prestress forces. If these are unsatisfactory for design, the prescribed parameters are appropriately adjusted and the system is reanalyzed for a shape and prestress that satisfies equilibrium. The iterative process continues until a satisfactory design solution is achieved for both the initial shape and a corresponding level of prestress force. However, a major problem is the lack of an efficient analysis method for the equilibrium configuration.

Argyris and Scharpf (1972) presented one of the first full discussions of analysis of static, nonlinear prestressed cable roof networks based on the finite element displacement method. Their purpose was primarily to estimate the basic equilibrium state of networks under prestress and dead loads. They term this the "central" problem, and that is a telling characterization of the importance of the Phase I problem.* Simple truss elements were employed that possessed both an elastic and a geometric stiffness matrix. These matrices are derived in the following section of the present report.

In addition to the use of finite element technology, this paper also introduced the use of an iterative method for solving the nonlinear algebraic equations that resulted from the finite element spatial discretization of the cable network. The iterative method was developed from engineering intuition and was essentially a relaxation of the residual nodal force, i.e., gradual reduction of the difference between the internal force and external force at each node point in the network. Moreover, they showed that their physically derived iterative method was equivalent to the well known Newton-Raphson mathematical iterative procedure.

The Newton iteration method is a standard method for the solution of nonlinear algebraic systems. It may be used in conjunction with load incrementation, i.e., the external load is applied in increments, and iteration to achieve equilibrium according to a preset tolerance is conducted within each load increment. This is referred to as an incremental/iterative procedure. Argyris and Scharpf noted that no convergence problems occurred in their computational experience. They attributed this to the use of a high-precision computer with a 60-bit word length as well as good physical intuition in selecting an initial geometry approximately satisfying equilibrium. However, there is evidence that they carefully watched the progress of the computed solution, and employed ad hoc techniques to adjust the system in cases where, for example, (truss element) forces tended towards compression.

Their computer program was later extended to handle static in-service load conditions for the Phase II problem. The same incremental/iterative procedure was used. They determined solutions for uniform and nonuniform snow loads, uniform wind loads, and uniform temperature loads. The initial

*It is interesting to note that this work was borne out of a practical need for structural analysis and design of cable net roof systems for facilities in the 1972 Olympiad.

configuration was that of the equilibrium position of the structure under dead load and prestress force. In Argyris, et al. (1973), the dynamics of cable systems was addressed with attention given to the method of temporal integration for both linear and nonlinear systems.

It should be mentioned that there are also two nonlinear finite element approaches pertaining to the Phase II cable problem for the solution of nonlinear dynamic equations of motion that do not require the formation and solution of a discrete system of equations. One is to employ a direct temporal integration of the equations of motion and an explicit integration operator, such as the central difference operator. An algebraic system is never formed, and a direct, step-by-step march through time is used to solve the equations of motion directly. This is a direct integration method and it is explicated well in many works that discuss structural dynamics, for example see Bathe (1982). It is generally applicable to structures subjected to highly transient loads. The other method is to employ the modal analysis procedure typical of linear systems analysis. It too is a direct attack on the solution of the set of nonlinear differential equations of motion. This method, which may be called nonlinear mode superposition, has been studied by Morris (1975 and 1977) and Geschwindner (1981) to solve land-based cable systems. It is based on updating the eigensolutions as the system's nonlinearity changes. Others are developing a similar methodology which is based on updating the orthogonal Ritz or Lanczos vectors in reduced subspaces; see for example, Wilson, et al. (1982); Akkari (1983); and Mish, et al. (1985). The motivation for this method is generally to reduce the size of the nonlinear structural dynamics problem, while retaining the important response and behavior of the structural model.

Ocean Cable Systems. Ocean cable systems are especially difficult to analyze in the same sense as are aeroelastic tensioned fabric systems for land-based cable system applications. This is due to many factors, but the two most important factors are their nonlinear geometric response, which has been discussed above for cables in general and which is a problem in structural mechanics, and their nonlinear hydrodynamic or aerodynamic response, which is a problem in fluid mechanics. A finite element analysis procedure for the solution of ocean cable and aeroelastic tensioned systems that would couple the equations of motion of the structure and the fluid is currently insurmountable. Few approaches aimed at practical analysis of these systems consider this true fluid-structure interaction approach.

An important work in ocean cable systems is the Ph.D. dissertation by Webster (1976) for it recognized and deals directly with the severe nonlinearity of ocean cable systems. Further, he organizes a general purpose solution approach for these systems around three-dimensional, nonlinear finite element technology.

Webster discusses large deflections occurring in ocean cable systems, and states that the dynamic response must be referenced to the deformed configuration of static equilibrium. Though he does not label it as such, implied in Webster's approach is the two-phase problem for cable analysis. The first phase is the establishment of a static equilibrium configuration due to the cable's prestress force and weight in water under quiescent conditions. The second phase is the calculation of large deflections that occur relative to the static equilibrium configuration due to in-service loads such as the dynamic forces of either steady or unsteady flows induced by waves and currents.

The practical computational effect of this is that the stiffness matrix for the cable structure becomes a function of displacements and, therefore, becomes nonlinear. However, in ocean cable systems displacements and spans are extremely large, and the geometric nonlinearity is perhaps more onerous in this class of problems than in any other in structural engineering. Further, very low tension states often exist causing the stiffness matrix to become nearly singular and to behave pathologically.

External hydrodynamic loads are difficult to describe mathematically due to a severely unpredictable environment. Once again tractability intervenes and a simplified, deterministic flow theory can be assumed. Then a generalized version of the Morison equation can be applied that includes a drag force term that is due to relative tangential flow along the cable, as well as the typical inertia and drag force terms that are due to relative normal flow. Further, these forces are path dependent and, therefore, nonconservative. The practical computational impact of this is that the external force vector in the system of equilibrium equations is nonlinear since it depends on the instantaneous cable configuration.

The phenomenon of vortex-induced response of marine cables is discussed by Griffin, et al. (1981) and Griffin (1982). This phenomena requires further research where design measures for mitigation of strumming are not effective. Further research is also required to describe the hydrodynamic loads generated on large, rigid bodies such as ships, platforms, and buoys that are moored using cable systems. These loads constitute the Phase II excitation for the mooring system's nonlinear dynamic response. Palo and Owens (1982) and Wu (1984) consider current-induced loads on moored vessels. These loads are considered important components in the overall consideration of cable system analysis because in actuality the hydrodynamic excitation on the hulls of moored vessels and the structural cable response of the mooring system are coupled hydroelastically.

Tensioned Fabric Systems. From the viewpoint of structural behavior, tensioned fabric structures are the two-dimensional analogue of cable structures. The Phase I and Phase II problems are conceptually the same. The spans involved with tensioned fabric structures are not as large as those for ocean cable systems, but the sag ratios are roughly equivalent, so the strength of the kinematic nonlinearity can be very similar. Much computational effort goes into obtaining the initial configuration solution of a tensioned fabric structure, see for example Haber and Abel (1982) and Hsu (1984).

Another related observation in computational aspects of tensioned fabric structures is that even when using special purpose engineering analysis software for these systems, often an accurate Phase I solution is ignored. In Shugar, et al., (1985) the initial displacement configuration was obtained by the Phase I static solution algorithm (Newton), but the prescribed, corresponding prestress conditions were assumed, and therefore could not have borne any relationship to the prestress conditions naturally occurring in the actual structure.* Though the prestress condition is important information, it remained unknown. The difficulty is that the software employed required the prescription of an assumed prestress level. It would then directly proceed to calculate a corresponding initial configuration by invoking the conditions of static equilibrium. Instead, what is required is a capability to use prescribed parameters such as required spans and the geometry of the unstrained fabric pattern, and then employ a Phase I solution algorithm robust enough to calculate both the initial configuration and the corresponding prestress due to dead load.

NONLINEAR CABLE FINITE ELEMENT MATRIX

The two-dimensional, static, nonlinear computer programs written for this investigation were based on a simple two-node truss element. This element is similar to the standard element used in the nonlinear finite element program SEADYN (Webster and Palo, 1982), which is used for much of the structural analysis of ocean cable systems in the Navy. The element's implementation, in this study, allows for compression forces to develop. This decision was a matter of expediency. A true cable element, perhaps, should provide for tension only behavior.

In the following, the linear elastic and geometric stiffness matrices for the truss element are developed and superimposed to get the nonlinear element stiffness matrix. The development follows that of Argyris and Scharpf (1972), and the notation follows that of Gallagher (1975).

*Sometimes an approximate Phase I solution is nonetheless sufficient as a starting configuration for the Phase II solution particularly when the stresses of the latter are substantially greater than the stresses of the former. However, accurate knowledge of the initial configuration and prestress is often important.

Cable Element Linear Elastic Stiffness Matrix Development

The column matrices $\underset{\sim}{x}^{t}$ and $\underset{\sim}{x}^{t+\Delta t}$ denote the positions of the cable element in the t configuration and the t+Δt configuration, respectively, as shown in Figure 1. They are defined as follows,

$$\underset{\sim}{x}^{t} = \left\lfloor x_1^t,\ y_1^t,\ z_1^t,\ x_2^t,\ y_2^t,\ z_2^t \right\rfloor^T$$

$$\underset{\sim}{x}^{t+\Delta t} = \left\lfloor x_1^{t+\Delta t},\ y_1^{t+\Delta t},\ z_1^{t+\Delta t},\ x_2^{t+\Delta t},\ y_2^{t+\Delta t},\ z_2^{t+\Delta t} \right\rfloor^T$$

They are related by the displacement matrix $\underset{\sim}{\Delta}$ as follows,

$$\underset{\sim}{x}^{t+\Delta t} = \underset{\sim}{x}^{t} + \underset{\sim}{\Delta}$$

where $\underset{\sim}{\Delta} = \left\lfloor \underset{\sim}{\Delta}_1^T,\ \underset{\sim}{\Delta}_2^T \right\rfloor^T$

$$= \left\lfloor u_1,\ v_1,\ w_1,\ u_2,\ v_2,\ w_2 \right\rfloor^T$$

The u_i, v_i, and w_i are Cartesian components of the nodal point displacement vectors $\boldsymbol{\Delta}_1$ and $\boldsymbol{\Delta}_2$ (see Figure 1) in the xyz system. That is,

$$\boldsymbol{\Delta}_1 = u_1\hat{i} + v_1\hat{j} + w_1\hat{k}$$

$$\boldsymbol{\Delta}_2 = u_2\hat{i} + v_2\hat{j} + w_2\hat{k}$$

The nodal point displacement components in the local element coordinate system x' are u_1' and u_2'. These displacements are axial and contribute directly to the axial deformation of the cable element. They are related to the global displacement components u_i, v_i, and w_i, shown in Figure 2, as follows,

$$u_1' = \boldsymbol{\Delta}_1 \cdot \hat{c}$$

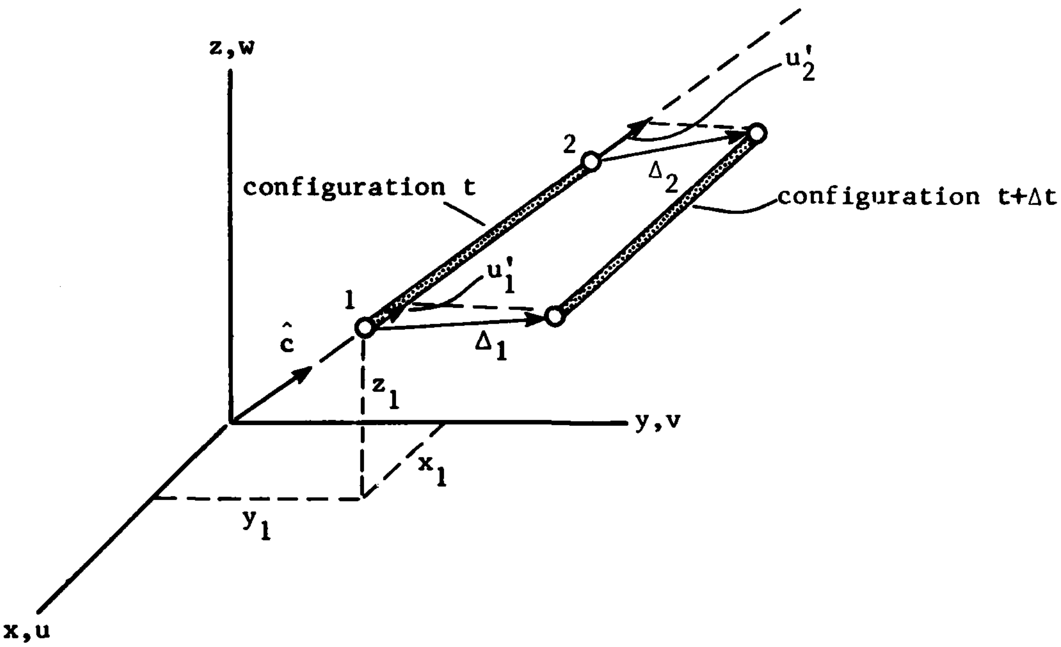


Figure 1. Displacement of a cable element.

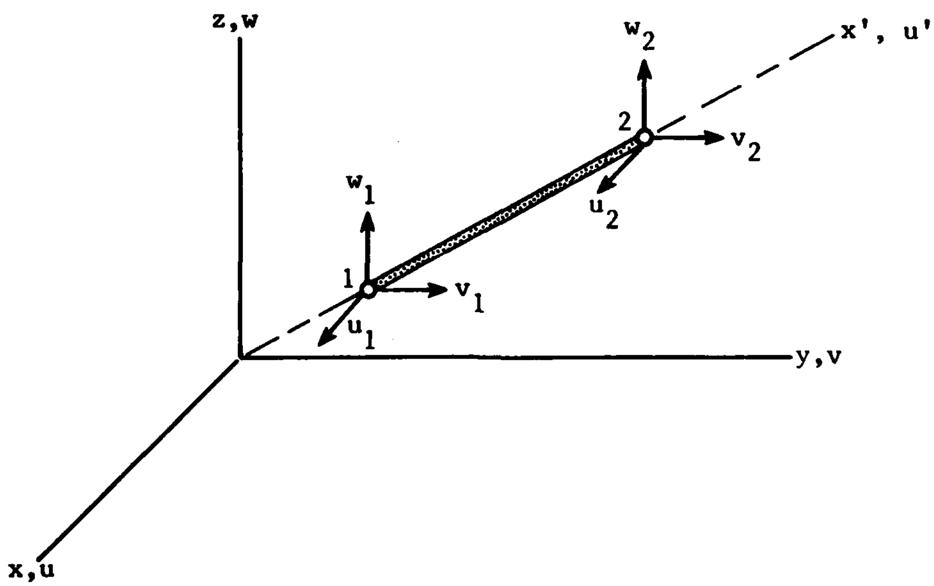


Figure 2. Global and local coordinate systems.

$$u_2' = \Delta_2 \cdot \hat{c}$$

where $\hat{c}$ is a unit vector along this x'-axis. That is,

$$\hat{c} = \ell_{x'x}\hat{i} + \ell_{x'y}\hat{j} + \ell_{x'z}\hat{k}$$

where $\ell_{x'x}$, $\ell_{x'y}$, and $\ell_{x'z}$ are the direction cosines of the cable element axis x' with respect to the x, y, and z axes, respectively. The direction cosines are computed as follows,

$$\ell_{x'x} = \frac{x_2 - x_1}{L}$$

$$\ell_{x'y} = \frac{y_2 - y_1}{L}$$

$$\ell_{x'z} = \frac{z_2 - z_1}{L}$$

where L is the (current, t+Δt) length of the element and is defined by:

$$L^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

Thus, the global to local transformation of displacements is described by,

$$\begin{Bmatrix} u_1' \\ u_2' \end{Bmatrix} = \begin{bmatrix} \ell_{x'x} & \ell_{x'y} & \ell_{x'z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \ell_{x'x} & \ell_{x'y} & \ell_{x'z} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix}$$

or,

$$\underset{\sim}{\Delta}' = \underset{\sim}{\Gamma}\,\underset{\sim}{\Delta}$$

The local and global nodal point forces are depicted in Figure 3. In matrix form, they are, respectively,

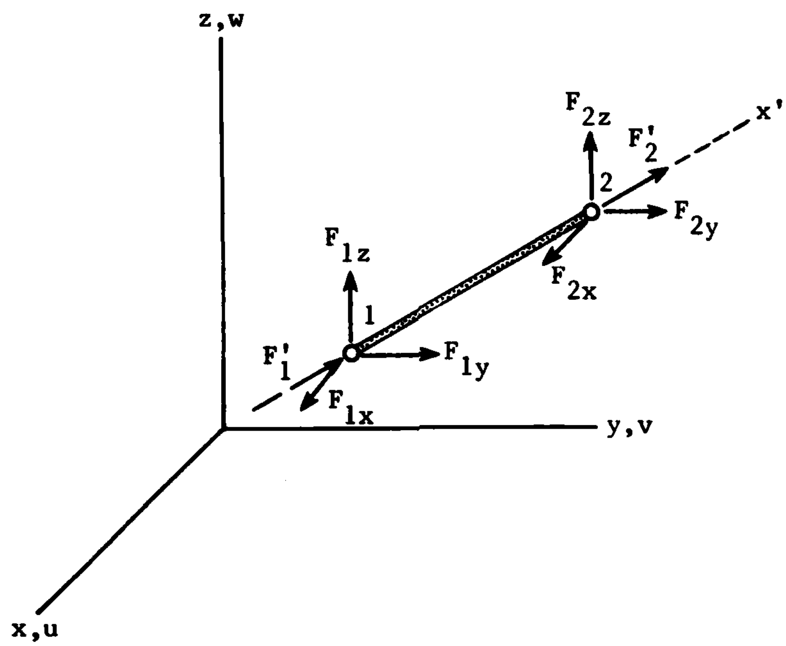


Figure 3. Global and local nodal forces.

$$(\underset{\sim}{F}')^T = \lfloor F_1', F_2' \rfloor^T$$

and,

$$(\underset{\sim}{F})^T = \lfloor F_{1x}, F_{1y}, F_{1z}, F_{2x}, F_{2y}, F_{2z} \rfloor^T$$

The transformation of nodal point forces from the local x'-axis to the global x, y, and z axes, is obtained as follows. The invariance of work under coordinate transformation requires that,

$$\frac{1}{2} (\underset{\sim}{F}')^T \underset{\sim}{\Delta}' = \frac{1}{2} \underset{\sim}{F}^T \underset{\sim}{\Delta}$$

It follows that,

$$(\underset{\sim}{F}')^T \underset{\sim}{T} \underset{\sim}{\Delta} = \underset{\sim}{F}^T \underset{\sim}{\Delta}$$

and since $\underset{\sim}{\Delta}$ is arbitrary,

$$\underset{\sim}{F}^T = (\underset{\sim}{F}')^T \underset{\sim}{T}$$

$$\underset{\sim}{F} = \underset{\sim}{I}^T \underset{\sim}{F}'$$

or,

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{1z} \\ F_{2x} \\ F_{2y} \\ F_{2z} \end{Bmatrix} = \begin{bmatrix} \ell_{x'x} & 0 \\ \ell_{x'y} & 0 \\ \ell_{x'z} & 0 \\ 0 & \ell_{x'x} \\ 0 & \ell_{x'y} \\ 0 & \ell_{x'z} \end{bmatrix} \begin{Bmatrix} F'_1 \\ F'_2 \end{Bmatrix}$$

Thus, the force transformation is implied by the displacement transformation.

In vector notation the force transformation is,

$$F'_1 = F_{1x}\,\hat{i} + F_{1y}\,\hat{j} + F_{1z}\,\hat{k}$$

$$F'_2 = F_{2x}\,\hat{i} + F_{2y}\,\hat{j} + F_{2z}\,\hat{k}$$

since, for example,

$$\begin{aligned} F_{1x} &= \ell_{x'x}\,F'_1 \\ &= F'_1 \cdot \hat{i} \end{aligned}$$

The nodal point forces components in the global system are the conventional rectangular components of the nodal point force vector in the local system. It is interesting to note that the same relationship is not true for the nodal point displacement transformation.

The force-displacement relationship in the local coordinate system is,

$$\underset{\sim}{F}' = \underset{\sim}{k}'_E \, \underset{\sim}{\Delta}'$$

where $\underset{\sim}{k}'_E = \frac{AE}{L_o} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

and is called the element elastic stiffness matrix. The symbols A and E denote the cable cross section area and Young's modulus, respectively, and L_o denotes the unstrained element length.

The element elastic stiffness matrix referred to the global system is denoted by $\underset{\sim}{k}_E$, and relates the forces and displacements in th xyz system as follows

$$\underset{\sim}{F} = \underset{\sim}{k}_E \underset{\sim}{\Delta}$$

This stiffness matrix is defined by the condition that the work associated with either conservative force $\underset{\sim}{F}$ or $\underset{\sim}{F}'$ is invariant under the coordinate axis transformation. Thus,

$$\frac{1}{2} (\underset{\sim}{F}')^T \underset{\sim}{\Delta}' = \frac{1}{2} \underset{\sim}{F}^T \underset{\sim}{\Delta}$$

It follows that,

$$(\underset{\sim}{k}'_E \underset{\sim}{\Delta}')^T \underset{\sim}{\Gamma} \underset{\sim}{\Delta} = (\underset{\sim}{k}_E \underset{\sim}{\Delta})^T \underset{\sim}{\Delta}$$

$$\underset{\sim}{k}_E \underset{\sim}{\Delta} = \underset{\sim}{\Gamma}^T \underset{\sim}{k}'_E \underset{\sim}{\Delta}' = \underset{\sim}{\Gamma}^T \underset{\sim}{k}'_E \underset{\sim}{\Gamma} \underset{\sim}{\Delta}$$

and, since $\underset{\sim}{\Delta}$ is arbitrary,

$$\underset{\sim}{k}_E = \underset{\sim}{\Gamma}^T \underset{\sim}{k}'_E \underset{\sim}{\Gamma}$$

This is a congruent transformation, and therefore, since $\underset{\sim}{k}'_E$ is a symmetric matrix, the matrix $\underset{\sim}{k}_E$ is also a symmetric matrix.

If the transformation matrix $\underset{\sim}{\Gamma}$ is partitioned as follows,

$$\underset{\sim}{\Gamma} = \left[\begin{array}{c|c} \underset{\sim}{\Gamma}_1 & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{\Gamma}_1 \end{array}\right]$$

it can be shown that the transformed stiffness matrix is:

$$\underset{\sim}{k}_E = \frac{AE}{L_o} \left[\begin{array}{c|c} \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 & -\underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 \\ \hline -\underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 & \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 \end{array}\right]$$

where the outer product $\underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1$ is the symmetric, 3 by 3 matrix,

$$\underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 = \begin{bmatrix} \ell_{x'x}^2 & \ell_{x'x}\ell_{x'y} & \ell_{x'x}\ell_{x'z} \\ \ell_{x'y}\ell_{x'x} & \ell_{x'y}^2 & \ell_{x'y}\ell_{x'z} \\ \ell_{x'z}\ell_{x'x} & \ell_{x'z}\ell_{x'y} & \ell_{x'z}^2 \end{bmatrix}$$

Geometric Stiffness Matrix Development

The following development brings in the contribution of a cable's prestress force to the cable element's total stiffness. It provides the resistance to lateral forces externally applied to a cable.

The displacement vectors Δ_1 and Δ_2 may be resolved into components parallel and perpendicular to the member axis x', as follows. For node i (i=1,2),

$$\Delta_i = \Delta_{i_{par}} + \Delta_{i_{per}}$$

Now,

$$\Delta_{i_{par}} = (\Delta_i \cdot \hat{c})\,\hat{c}$$

In matrix form this can be shown to expand to:

$$\begin{Bmatrix} \Delta_{ix} \\ \Delta_{iy} \\ \Delta_{iz} \end{Bmatrix}_{par} = \begin{bmatrix} \ell_{x'x}^2 & \ell_{x'x}\ell_{x'y} & \ell_{x'x}\ell_{x'z} \\ \ell_{x'y}\ell_{x'x} & \ell_{x'y}^2 & \ell_{x'y}\ell_{x'z} \\ \ell_{x'z}\ell_{x'x} & \ell_{x'z}\ell_{x'y} & \ell_{x'z}^2 \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix}$$

or,

$$\underset{\sim}{\Delta}_{i_{par}} = \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 \underset{\sim}{\Delta}_i$$

The perpendicular component is then:

$$\Delta_{i_{per}} = \Delta_i - \Delta_{i_{par}}$$

or, in matrix form:

$$
\begin{aligned}
\underset{\sim}{\Delta}_{i_{per}} &= \underset{\sim}{\Delta}_i - \underset{\sim}{\Delta}_{i_{par}} \\
&= \underset{\sim}{\Delta}_i - \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 \underset{\sim}{\Delta}_i \\
&= \underset{\sim}{I} \underset{\sim}{\Delta}_i - \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 \underset{\sim}{\Delta}_i \\
&= (\underset{\sim}{I} - \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1) \underset{\sim}{\Delta}_i
\end{aligned}
$$

where $\underset{\sim}{I}$ is the 3 by 3 identity matrix.

A measure of the rotation of the cable element is given by the difference in the perpendicular components as follows,

$$\bar{\delta} = \Delta_{2_{per}} - \Delta_{1_{per}}$$

In matrix form this is expressed as:

$$
\begin{aligned}
\underset{\sim}{\bar{\delta}} &= \underset{\sim}{\Delta}_{2_{per}} - \underset{\sim}{\Delta}_{1_{per}} \\
&= (\underset{\sim}{I} - \underset{\sim}{\Gamma}^T \underset{\sim}{\Gamma})(\underset{\sim}{\Delta}_2 - \underset{\sim}{\Delta}_1)
\end{aligned}
$$

Perpendicular forces f_G at each node point are developed to equilibrate the couple formed during the member rotation by the axial force f, which is the prestress force existing in the member, as shown in Figure 4. These nodal point forces are computed in terms of the axial force and member rotation as follows,

$$f_{G1} = -\frac{f}{L_o}\bar{\delta} \qquad \text{and} \qquad f_{G2} = \frac{f}{L_o}\bar{\delta}$$

In matrix notation this becomes:

$$\begin{bmatrix} \underset{\sim}{f}_{G1} \\ \underset{\sim}{f}_{G2} \end{bmatrix} = \frac{f}{L_o} \begin{bmatrix} -\underset{\sim}{\bar{\delta}} \\ \underset{\sim}{\bar{\delta}} \end{bmatrix}$$

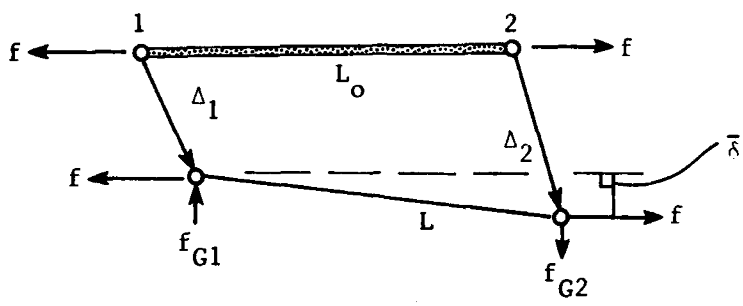


Figure 4. Formation of geometric forces.

$$= \frac{f}{L_o} \begin{bmatrix} (\mathbf{I} - \mathbf{\Gamma}_1^T \mathbf{\Gamma}_1) & -(\mathbf{I} - \mathbf{\Gamma}_1^T \mathbf{\Gamma}) \\ -(\mathbf{I} - \mathbf{\Gamma}^T \mathbf{\Gamma}) & (\mathbf{I} - \mathbf{\Gamma}_1^T \mathbf{\Gamma}_1) \end{bmatrix} \begin{bmatrix} \mathbf{\Delta}_1 \\ \mathbf{\Delta}_2 \end{bmatrix}$$

or,

$$\mathbf{f}_G = \mathbf{k}_G \, \mathbf{\Delta}$$

The matrix $\mathbf{k}_G$ is known as the geometric stiffness matrix. It is completely independent of the element's elastic properties, and depends only on the element's geometrical property L_o.

Total Cable Element Stiffness Matrix

The total cable element stiffness is obtained from:

$$\mathbf{F} = \mathbf{F}_E + \mathbf{f}_G = \mathbf{k}_E \, \mathbf{\Delta} + \mathbf{k}_G \, \mathbf{\Delta} = \mathbf{k} \, \mathbf{\Delta}$$

Thus, the total stiffness matrix $\mathbf{k}$ is the sum of the elastic and geometric stiffness matrices. It can be written in a computationally organized manner as follows:

$$\underset{\sim}{k} = \underset{\sim}{k}_E + \underset{\sim}{k}_G = \begin{bmatrix} \underset{\sim}{k}' & -\underset{\sim}{k}' \\ -\underset{\sim}{k}' & \underset{\sim}{k} \end{bmatrix}$$

Here, the submatrix $\underset{\sim}{k}'$ is defined as:

$$\underset{\sim}{k}' = \left(\frac{AE - f}{L_o}\right) \underset{\sim}{\Gamma}_1^T \underset{\sim}{\Gamma}_1 + \frac{f}{L_o} \underset{\sim}{I}$$

The elements of $\underset{\sim}{k}'$ are listed in Table 1.

Table 1. Coefficients of Submatrix $\underset{\sim}{k}'$.

$$\begin{bmatrix} a\,\ell_{x'x}^2 + b & a\,\ell_{x'x}\,\ell_{x'y} & a\,\ell_{x'x}\,\ell_{x'z} \\ & a\,\ell_{x'y}^2 + b & a\,\ell_{x'y}\,\ell_{x'z} \\ \text{symmetric} & & a\,\ell_{x'z}^2 + b \end{bmatrix}$$

$$\text{where} \quad a = \frac{AE - f}{L_o} \quad \text{and} \quad b = \frac{f}{L_o}$$

When a restraint against rigid body translation of the cable element, such as pinning one end, is imposed, it can be shown that,

$$\det \underset{\sim}{k} = \det (\underset{\sim}{k}_E + \underset{\sim}{k}_G) \neq 0 \text{ for } f \neq 0$$

and,

$$\det \underset{\sim}{k} = \det \underset{\sim}{k}_E = 0 \text{ for } f = 0$$

That is, the pinned element will resist rotation only when the prestress force f is nonzero.

FULL NEWTON AND MODIFIED NEWTON SOLUTION METHODS

Standard approaches in nonlinear finite element technology consider the formation and solution of a nonlinear system of discrete finite element equations, i.e., an algebraic system. The form of these equations is the same for all nonlinear kinematic structures that have been spatially subdivided by the displacement-based finite element method. The approach is viable for both phases of the two-phase cable problem. There are no differences in the form of the equations that would arise from the use of cable finite elements compared to other finite elements in non-cable problems, i.e., continuum or structural elements. The formal derivation of these equations can be lengthy and the reader is, therefore, deferred to standard treatments such as the one by Bathe (1982). Only a summary is given here to introduce the status quo solution methods, the full and modified Newton methods.

The derivation of the discrete nonlinear finite element equations is ordinarily based on the principle of virtual work applied to nonlinear geometric bodies that undergo large displacements, large rotations, and small strains. Displacements and rotations of cables or cable segments are large, but cable strains are assumed small with no change in cable cross-sectional area. Typically, a material formulation is used, which may be either a total Lagrangian or an updated Lagrangian formulation. In this method the motion of the cable is followed from its initial to its final configuration in an incremental fashion. In the total Lagrangian formulation, all variables corresponding to the deformed configuration are referred to the initial configuration. In the updated Lagrangian formulation, the variables are referred to the most previously calculated configuration. Bathe (1982) states that the two formulations are equivalently comprehensive in their inclusion of nonlinear effects, and will yield identical results if consistently carried out. Preference for either formulation is based on implementation and computational considerations.

The objective in nonlinear analysis of cable systems is to first find the equilibrium configuration corresponding to the applied load. Most often an incremental approach to the solution is used wherein the equilibrium state is established via a solution of the nonlinear equation system for a succession of applied load increments, until finally the total load has been applied to the cable system. For any given configuration the equilibrium equations are represented as:

$$\underset{\sim}{F}(\underset{\sim}{x}) = \underset{\sim}{F}_e - \underset{\sim}{F}_i = \underset{\sim}{0}$$

where: $\underset{\sim}{F}_e$ = vector of external nodal point forces due to the applied loads corresponding to the present configuration

$\underset{\sim}{F}_i$ = vector of internal nodal point forces due to the element stresses corresponding to the present configuration

$\underset{\sim}{x}$ = vector of displacement variables constituting the system configuration

These are the discrete nonlinear finite element equations. The displacement vector, $\underset{\sim}{x}$, constitutes the solution to the equilibrium configuration when the residual force vector, $\underset{\sim}{F}(\underset{\sim}{x})$, is zero. Satisfying these equations is the objective of many different nonlinear finite element solution algorithms.

Full Newton Method

We have said that application of the finite element method to any nonlinear structure, and further application of a time-stepping procedure in the case of dynamic problems, leads to a set of nonlinear algebraic equations for each discrete time. These discrete nonlinear equations are represented by:

$$\underset{\sim}{F}(\underset{\sim}{x}) = \underset{\sim}{0}$$

If we write a first order Taylor expansion we obtain the equation

$$\underset{\sim}{F}(\underset{\sim}{x}_{k+1}) \simeq \underset{\sim}{F}(\underset{\sim}{x}_k) + \underset{\sim}{F}'(\underset{\sim}{x}_k)\, \underset{\sim}{d}_k$$

where k is the iterate number, $\underset{\sim}{x}_k$ is a previously computed displacement vector, $\underset{\sim}{d}_k$ is a change in $\underset{\sim}{x}_k$ called the step direction and is an unknown, and $\underset{\sim}{F}'$ is the Jacobian (or tangent stiffness matrix) of $\underset{\sim}{F}$ whose elements are defined by:

$$F'_{ij} = \frac{\partial F_i}{\partial x_j}$$

In one-dimension, these quantities appear as in Figure 5.

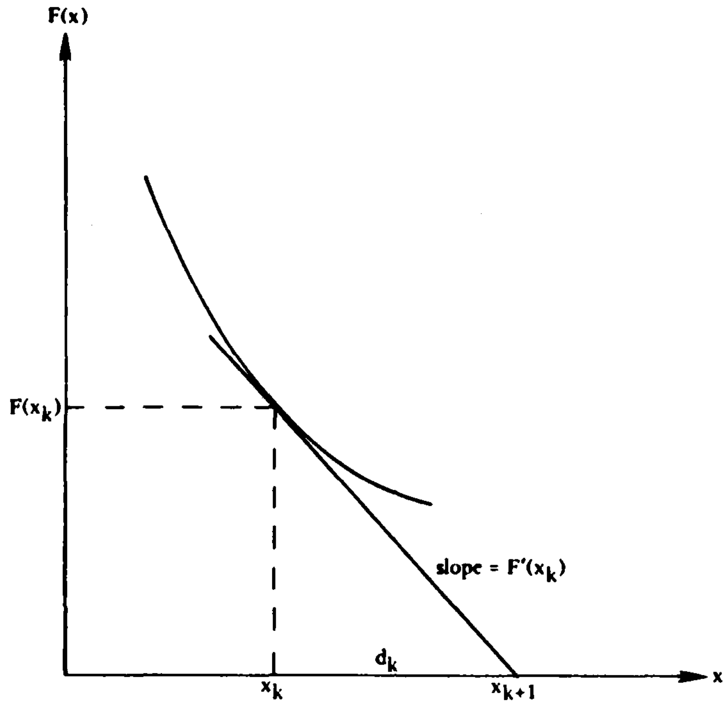


Figure 5. The tangent stiffness - Newton's Method.

Newton's method consists of setting the Taylor expansion to zero, solving for $\underset{\sim}{d}_k$ in the equation:

$$\underset{\sim}{F}'(\underset{\sim}{x}_k)\ \underset{\sim}{d}_k = -\underset{\sim}{F}(\underset{\sim}{x}_k)$$

and setting:

$$\underset{\sim}{x}_{k+1} = \underset{\sim}{x}_k + \underset{\sim}{d}_k$$

to obtain the updated configuration.

In practice it is often desirable to modify the last formula as follows

$$\underset{\sim}{x}_{k+1} = \underset{\sim}{x}_k + s_k\ \underset{\sim}{d}_k$$

where s_k is a scalar used to enhance the stability of the algorithm. The value of s_k is determined from a line search. That is, it is determined in such a way as to minimize the residual, $\underset{\sim}{F}(\underset{\sim}{x}_k)$. A common line search procedure is to solve the equation:

$$d_k^T F(x_k + s_k d_k) = 0$$

for s_k.

Algorithm 1 is a procedural description of the full Newton method for solving a system of discrete nonlinear finite element equations.

Algorithm 1. The Full Newton Algorithm.

0. Set $k \leftarrow 0$ and initialize x_0 and k_{max}
1. Compute $F(x_k)$
2. Compute the tangent stiffness matrix $F'(x_k)$
3. Solve $F'(x_k)\, d_k = - F(x_k)$ for d_k
4. Compute s_k from a line search
5. Update $x_{k+1} \leftarrow x_k + s_k d_k$
6. Test for convergence or whether $k = k_{max}$
7. Terminate iteration or increment k and repeat steps (1) through (7)
8. Output x_{k+1} and stop

Newton's algorithm has at least two very desirable properties:

1. Any x_k in the domain of convergence results in an x_{k+1} that is also in the domain. Consequently, the method is stable and convergent once any iterate is in the domain of convergence.

2. The method possesses at least super-linear convergence and often quadratic convergence.

On the negative side, it has two disadvantages:

1. If the domain of convergence is small, then a very good initial approximation, x_0, to the solution vector is required.

2. Evaluations of the tangent stiffness matrix $\mathbf{F}'(\mathbf{x}_k)$ and its factorization (step 3 in Algorithm 1) are very costly in large finite element problems.

The requirement of a good initial guess may be mitigated by using line searches and, for quasi-static problems, by using an incremental evolution of the load application. In the next method, the possibility of reducing the computational effort in factorization of the tangent stiffness matrix is addressed.

Modified Newton Method

For large systems of equations, the main cost in Newton's method is the formation and factorization of the tangent stiffness matrix. To mitigate this cost, it is often advisable to use a previously computed and factored tangent stiffness matrix as an approximation of the current tangent stiffness matrix. This is indicated in step (2) of Algorithm 2 where $\mathbf{F}'(\mathbf{x}_i)$ represents a previously formed tangent stiffness matrix, and $\mathbf{B}_k$ represents its factored form. Such a method is called a modified Newton method.

Algorithm 2. The Modified Newton Algorithm

0. Set $k \leftarrow 0$ and initialize $\mathbf{x}_0$ and k_{max}
1. Compute $\mathbf{F}(\mathbf{x}_k)$
2. Solve $\mathbf{B}_k \mathbf{d}_k = -\mathbf{F}(\mathbf{x}_k)$, where $\mathbf{B}_k = \mathbf{F}'(\mathbf{x}_i)$; $i \leq k$
3. Compute s_k from a line search
4. Update $\mathbf{x}_{k+1} = \mathbf{x}_k + s_k \mathbf{d}_k$
5. Test for convergence or whether $k = k_{max}$
6. Terminate the iteration or increment k and repeat steps (1) through (6)
7. Output $\mathbf{x}_{k+1}$ and stop

For each loop through Algorithm 2 when the value of i in step (2) remains unchanged, the $O(n^3)$ steps required for factoring the tangent stiffness matrix, are avoided. However, these savings are achieved at the expense of a less satisfactory convergence rate. The modified Newton method only converges linearly, as indicated by the inequality:

$$||\underset{\sim}{x}_{k+1} - \underset{\sim}{x}^*|| \leq ||\alpha(\underset{\sim}{x}_k - \underset{\sim}{x}^*)||$$

where $\underset{\sim}{x}^*$ denotes the true solution vector, and α is a scalar between 0 and 1.

After a certain number, say p, of iterations through the modified Newton algorithm, the value of i in step (2) may be incremented, and a new tangent stiffness matrix must then be factored. However, the value of p is very problem-dependent and depends on the degree of nonlinearity of the cable system. It is reasonable to assume that p will be smallest for the more highly nonlinear cable systems. But, as the tautness and stiffness of the cable system increases, the value of p may be allowed to increase to reduce computational effort.

DERIVATION OF DYNAMIC RELAXATION INTEGRATION FORMULAS

The system of N static nonlinear equations to be solved for the unknown, N-dimensional displacement vector $\underset{\sim}{x}$ is, as before,

$$\underset{\sim}{F}_i(\underset{\sim}{x}) = \underset{\sim}{F}_e$$

where $\underset{\sim}{F}_i(\underset{\sim}{x})$ is the nonlinear internal force vector and $\underset{\sim}{F}_e$ is the applied external force vector.

The dynamic relaxation solution method begins by converting the static problem to a structural dynamics problem in terms of a pseudo time, t. Thus,

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{x}} + \underset{\sim}{C}\,\dot{\underset{\sim}{x}} + \underset{\sim}{F}_i(\underset{\sim}{x}) = \underset{\sim}{F}_e$$

is the pseudo equation of motion to be solved for $\underset{\sim}{x}$. Here $\underset{\sim}{M}$ and $\underset{\sim}{C}$ are, respectively, artificial mass and damping matrices which are defined arbitrarily as:

$$\underset{\sim}{M} = \rho \, \underset{\sim}{I}$$

$$\underset{\sim}{C} = c \, \underset{\sim}{I}$$

where $\underset{\sim}{I}$ is the NxN identity matrix, and the scalars ρ and c are the artificial mass and damping parameters. These parameters will subsequently be chosen to control stability and convergence of the integration procedure.

Temporal integration is accomplished by discretizing the dynamic equations using the following central difference approximation formulas, which are also illustrated in Figure 6.

$$\dot{\underset{\approx}{x}}^{n-1/2} = \frac{1}{h} (\underset{\approx}{x}^{n} - \underset{\approx}{x}^{n-1})$$

and,

$$\ddot{\underset{\approx}{x}}^{n} = \frac{1}{h} (\dot{\underset{\approx}{x}}^{n+1/2} - \dot{\underset{\approx}{x}}^{n-1/2})$$

where h is the time step size. Also, averaging the discrete velocities at n+1/2 and n-1/2 yields:

$$\dot{\underset{\approx}{x}}^{n} = \frac{1}{2} (\dot{\underset{\approx}{x}}^{n+1/2} + \dot{\underset{\approx}{x}}^{n-1/2})$$

Substituting these formulas into the pseudo dynamic equations of motion yields a two-term recursion formula for updating the velocity:

$$\dot{\underset{\approx}{x}}^{n+1/2} = \left(\frac{2 - ch/\rho}{2 + ch/\rho}\right) \dot{\underset{\approx}{x}}^{n-1/2} - \left(\frac{2\ h/\rho}{2 + ch/\rho}\right) \underset{\sim}{r}^{n}$$

where $\underset{\sim}{r}^{n}$ is the residual force vector and is defined as the difference between the internal and external force vectors,

$$\underset{\sim}{r}^{n} = \underset{\sim}{F}_{i}(\underset{\approx}{x}^{n}) - \underset{\sim}{F}_{e}(t^{n})$$

The displacement is updated by the formula:

$$\underset{\approx}{x}^{n+1} = \underset{\approx}{x}^{n} + h \, \dot{\underset{\approx}{x}}^{n+1/2}$$

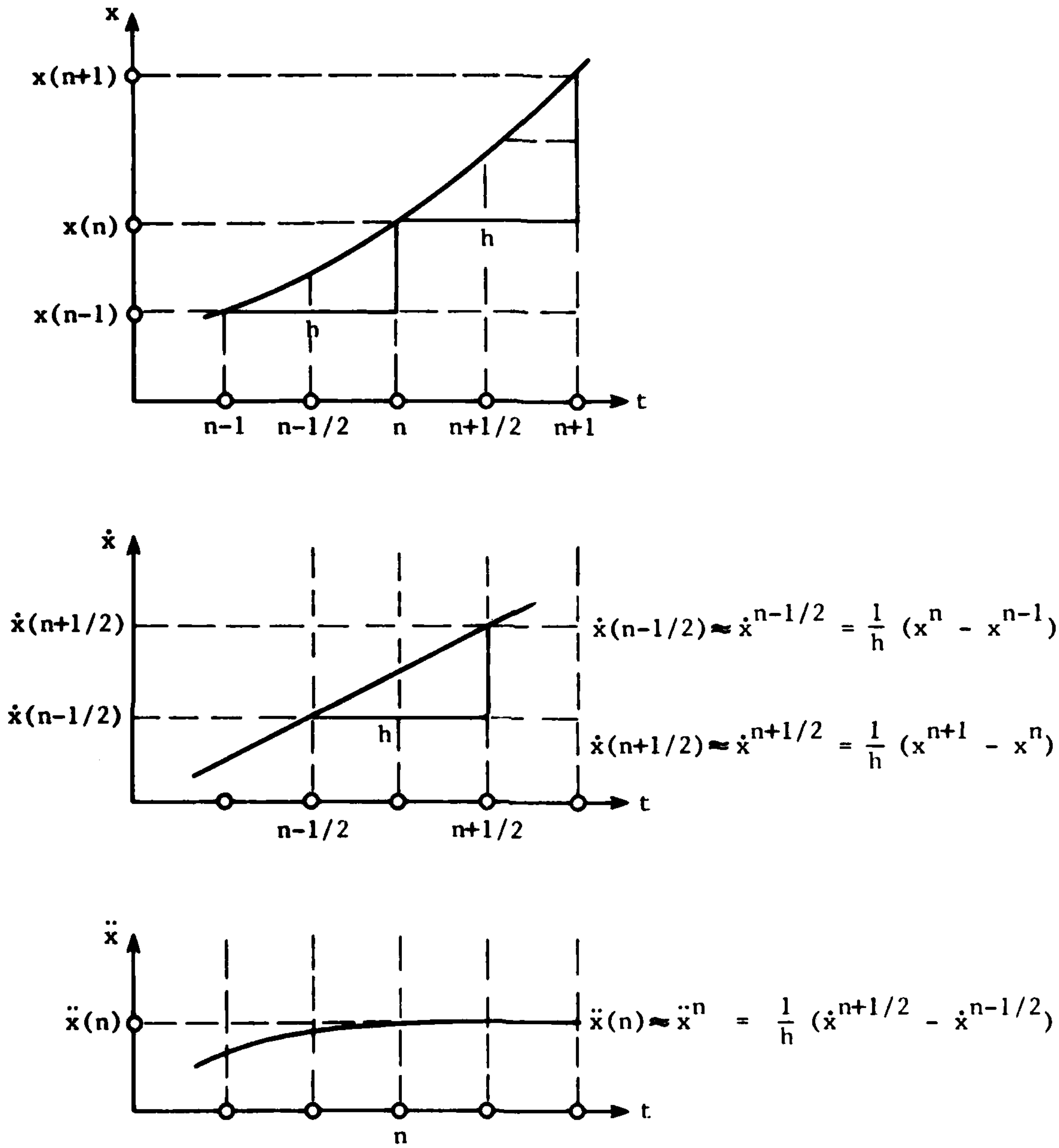


Figure 6. Central difference approximation.

which is obtained from the difference formula for velocity with n+1 replacing n. Combining these two updated formulas and using the difference equation for $\dot{\underset{\sim}{x}}^{n-1/2}$ (to eliminate $\dot{\underset{\sim}{x}}^{n-1/2}$) yields a single, three-term recursion updated formula (see Papadrakakis, 1982) for the displacement vector:

$$\underset{\sim}{x}^{n+1} = \left(\frac{4}{2 + ch/\rho}\right) \underset{\sim}{x}^{n} - \left(\frac{2 - ch/\rho}{2 + ch/\rho}\right) \underset{\sim}{x}^{n-1} - \left(\frac{2\, h^2/\rho}{2 + ch/\rho}\right) \underset{\sim}{r}^{n} \quad (n > 0)$$

This is the primary formula for the dynamic relaxation method. It does not apply for $n = 0$, for the displacement at $\underset{\sim}{x}^{-1}$ is unknown. Therefore, an additional formula is needed to start the integration.

The following initial conditions for displacement and velocity

$$\underset{\sim}{x}(0) = \underset{\sim}{x}^{0}$$

$$\dot{\underset{\sim}{x}}(0) = \dot{\underset{\sim}{x}}^{0}$$

are arbitrary since this is a pseudo dynamic formulation. We may, for example, choose:

$$\dot{\underset{\sim}{x}}^{0} = \underset{\sim}{0}$$

and retain $\underset{\sim}{x}^{0}$ as arbitrary. Next we use both the central difference and the average velocity formula with $n = 0$ to determine $\underset{\sim}{x}^{-1}$ as follows

$$\underset{\sim}{x}^{-1} = \underset{\sim}{x}^{0} - h\, \dot{\underset{\sim}{x}}^{-1/2} = \underset{\sim}{x}^{0} + h\, \dot{\underset{\sim}{x}}^{+1/2}$$

Then we may use the central difference formula (with $n = 1$) for $\dot{\underset{\sim}{x}}^{1/2}$ and get:

$$\underset{\sim}{x}^{-1} = \underset{\sim}{x}^{1}$$

Substituting this into the primary updated formula for displacement with $n = 0$, we arrive at:

$$\underset{\sim}{x}^{1} = \underset{\sim}{x}^{0} - \frac{h^2}{2\rho} \underset{\sim}{r}^{0}$$

This is the desired starting formula for the dynamic relaxation method. It is observed that this formula does not quite agree with the starting formula given by Papadrakakis (1981), which is:

$$\underset{\sim}{x}^{1} = \underset{\sim}{x}^{0} - \frac{h}{2\rho}\underset{\sim}{r}^{0}$$

unless h = 1. It does, however, agree with the starting formula given by Underwood (1983). However, the primary updated formula does agree with those given by both the above authors.

It should be emphasized that the calculation of the residual $\underset{\sim}{r}^{n}$ in the dynamic relaxation method is computationally straightforward. All that is required is to calculate and sum the unbalanced forces existing at each node point in the cable system at the known, current configuration $\underset{\sim}{x}^{n}$. The internal force calculation for each element is simply based on the linear portion of the element stiffness matrix. It is not necessary to use the nonlinear element stiffness matrix. So the residual force vector is calculated on the local level. It is not necessary to compute the tangent stiffness matrix of the system at the current configuration for the purpose of computing the internal force as the product $\underset{\sim}{K}(\underset{\sim}{x}^{n})\underset{\sim}{x}^{n}$, as is necessary for the Newton-based procedures.

A Generalization of the Method

A family of dynamic relaxation schemes may be derived by preconditioning the system of nonlinear equilibrium equations prior to their solution by dynamic relaxation. Papadrakakis (1986) demonstrates the preconditioned dynamic relaxation scheme for linear systems of equations. Here, we apply the same preconditioning scheme to the linearized form of the nonlinear system,

$$\underset{\sim}{K}(\underset{\sim}{x}^{n})\ \underset{\sim}{x}^{n} = \underset{\sim}{F}_{e}(\underset{\sim}{t}^{n})$$

Premultiplying this system with the inverse of the preconditioning matrix $\underset{\sim}{W}^{-1}$, we get*:

$$\underset{\sim}{W}^{-1}\ \underset{\sim}{K}(\underset{\sim}{x}^{n})\ \underset{\sim}{W}^{-T}\ \underset{\sim}{W}^{T}\ \ \underset{\sim}{x}^{n} = \underset{\sim}{W}^{-1}\ \underset{\sim}{F}_{e}(\underset{\sim}{t}^{n})$$

*The superscript -T means the inverse of the transpose of the matrix.

Thus, the general form of the residual, $\underset{\sim}{r}^n$, in the dynamic relaxation method may be written:

$$\underset{\sim}{r}^n = \underset{\sim}{W}^{-1}[\underset{\sim}{K}(\underset{\sim}{x}^n)\, \underset{\sim}{W}^{-T}\, \underset{\sim}{W}^T\, \underset{\sim}{x}^n - \underset{\sim}{F}_e\, (t^n)]$$

Alternative, preconditioning matrices may be defined as:

a. $\underset{\sim}{W} = \underset{\sim}{I}$. This is the straight dynamic relaxation scheme which has been presented above.

b. $\underset{\sim}{W} = \underset{\sim}{D}^{1/2}$. Here $\underset{\sim}{D}$ is a diagonal matrix composed of the entries on the main diagonal of the tangent stiffness matrix. This method is also called diagonal scaling since the original linearized system is scaled such that an equivalent system results in which the entries on the main diagonal of the coefficient matrix, formed from the tri-product matrix, $\underset{\sim}{W}^{-1}\underset{\sim}{K}(\underset{\sim}{x}^n)$ $\underset{\sim}{W}^{-T}$, are unity. Physically, this method is equivalent to assuming that the mass and damping matrices in the dynamic relaxation process are proportional to the diagonal stiffness matrix.

c. $\underset{\sim}{W} = (\underset{\sim}{D} - \omega\, \underset{\sim}{C}_L)\, \underset{\sim}{D}^{-1/2}$. This preconditioning matrix assumes the tangent stiffness matrix can be factored as:

$$\underset{\sim}{K}(\underset{\sim}{x}^n) = \underset{\sim}{D} - \underset{\sim}{C}_L - \underset{\sim}{C}_U$$

where $-\underset{\sim}{C}_L$ and $-\underset{\sim}{C}_U$ are the strictly lower and upper triangular matrices of $\underset{\sim}{K}$. The scalar ω is an acceleration parameter, which is usually set to unity.

DERIVATION OF OPTIMAL ITERATION PARAMETERS

Since the artificial inertia and damping forces appearing in the pseudo equations of motion are arbitrary, we are at liberty to choose any values for the parameters ρ and c. The objective here is to develop formulas for these parameters so that they may be adaptively controlled (during the solution) in such a way as to promote optimum stability and convergence for the dynamic relaxation iterative procedure. We assume that any difficulty in achieving stability and convergence would be due to the numerically ill-conditioned stiffness matrix $\underset{\sim}{K}(\underset{\sim}{x})$. Such ill-conditioning may arise for valid physical reasons, such as when cables approach slack conditions.

The internal restoring force term may be represented as*:

$$\underset{\sim}{F}_i(\underset{\sim}{x}) = \underset{\sim}{K}(\underset{\sim}{x})\ \underset{\sim}{x}$$

Thus, whatever the desired optimizing formulas are, it makes sense that they should reflect the condition of the stiffness matrix during the solution process.

To examine systematically the stability and convergence of the method, Lynch (1968) transformed the iterative process into a standard eigenvalue problem for the error vector:

$$\underset{\sim}{\varepsilon}^n = \underset{\sim}{x}^n - \underset{\sim}{x}$$

where $\underset{\sim}{x}$ denotes the exact solution for the displacement vector. Following this approach, the relationship between successive error vectors is given by:

$$\underset{\sim}{\varepsilon}^{n+1} = [\beta\ \underset{\sim}{I} - \gamma\ \underset{\sim}{B}]\ \underset{\sim}{\varepsilon}^n - \alpha\ \underset{\sim}{\varepsilon}^{n-1}$$

where: $\beta = \dfrac{2 - ch/\rho}{2 + ch/\rho} + 1$

*Recall, however, that the internal restoring force is actually calculated independently of this expression.

$$\gamma = \frac{2\ h^2/\rho}{2 + ch/\rho}$$

$$\alpha = \beta - 1$$

The matrix $\underset{\sim}{B}$ is the preconditioned tangent stiffness matrix,

$$\underset{\sim}{B} = \underset{\sim}{W}^{-1}\ \underset{\sim}{K}(\underset{\sim}{x}^n)\ \underset{\sim}{W}^{-T}$$

Letting the rate at which the error vector decays be denoted by λ, then*:

$$\underset{\sim}{\varepsilon}^{n+1} = \lambda\ \underset{\sim}{\varepsilon}^n$$

We wish to investigate what influences the decay rate λ. So an equation for λ is obtained by substituting the above definition into Lynch's equation. We can then obtain:

$$\left[\left(\frac{\lambda^2 - \beta\ \lambda + \alpha}{\gamma\ \lambda}\right) \underset{\sim}{I} + \underset{\sim}{B}\right] \underset{\sim}{\varepsilon}^n = \underset{\sim}{0}$$

Now, the standard eigenvalue problem for any eigenvalue λ_B of the matrix $\underset{\sim}{B}$ is written as:

$$(-\ \lambda_B\ \underset{\sim}{I} + \underset{\sim}{B})\ \underset{\sim}{\varepsilon}_B = \underset{\sim}{0}$$

If we identify this equation with the one above it, we obtain:

$$\frac{\lambda_{DR}^2 - \lambda_{DR}\ \beta + \alpha}{\lambda_{DR}\ \gamma} + \lambda_B = 0$$

Here we glimpse that the decay rate λ_{DR} is related to an eigenvalue λ_B of the stiffness matrix (scaled). This is the type of relationship that was anticipated, i.e., the stiffness matrix conditioning is tracked by monitoring an eigenvalue.

*It's apparently an assumption that the iteration process converges monitonically, and that the decay rate is a constant, scalar multiplier of the previous error vector.

Solving for λ_{DR} we obtain the quadratic equation:

$$\lambda_{DR}^2 - (\beta - \gamma \lambda_B) \lambda_{DR} + \alpha = 0$$

So, for each of the N eigenvalues of $\underset{\sim}{B}$ there are two solutions for λ_{DR},

$$\lambda_{DR} = \frac{1}{2} (\beta - \gamma \lambda_B) \pm \frac{1}{2} \sqrt{(\beta - \gamma \lambda_B)^2 - 4 \alpha}$$

In the first case, the roots of the quadratic are complex when:

$$4 \alpha > (\beta - \gamma \lambda_B)^2$$

The modulus of the decay rate $|\lambda_{DR}|$ can be found and is:

$$|\lambda_{DR}| = \sqrt{\frac{2 - ch/\rho}{2 + ch/\rho}}$$

The decay rate modulus is seen to be independent of the eigenvalue λ_B in the case of complex roots.

In the second case, the roots are real and equal when:

$$4 \alpha = (\beta - \gamma \lambda_B)^2$$

The roots are:

$$\lambda_{DR} = \frac{1}{2 + ch/\rho} \left(2 - \frac{h^2}{\rho} \lambda_B\right)$$

For this case, we also find that the square of the convergence parameter ch/ρ has the following relationship with λ_B

$$\left(\frac{ch}{\rho}\right)^2 = \frac{\lambda_B h^2}{\rho} \left(4 - \frac{\lambda_B h^2}{\rho}\right)$$

In the third case, the roots are real and unequal when:

$$4 \alpha < (\beta - \gamma \lambda_B)^2$$

The larger of the two roots is found to be:

$$\lambda_{DR} = \frac{1}{2 + ch/\rho}\left[\left(2 - \frac{h^2 \lambda_B}{\rho}\right) + \sqrt{\frac{\lambda_B^2 h^4}{\rho^2} - \frac{4 \lambda_B h^2}{\rho} + \frac{c^2 h^2}{\rho^2}}\right]$$

From the definition of λ, we see that it is important to promote the smallest value possible for this parameter. Comparing the formulas for λ_{DR} in the three cases above, we find that the smallest value will occur for the case of equal roots. In Figure 7, we graph the formula for λ_{DR} for this case against the parameter $h^2\lambda_B/\rho$ while holding constant the parameter ch/ρ. We also note that $h^2\lambda_B/\rho \leq 4$, since this would otherwise lead to imaginary values for the parameter ch/ρ, which has been identified as real in the dynamic relaxation formulation. Thus whatever the N values of λ_B are, we must choose h^2/ρ so that the range

$$0 < \frac{h^2}{\rho}\lambda_B < 4$$

is maintained.

At this point we introduce the prospect of calculating the minimum and maximum eigenvalues, λ_{Bmax} and λ_{Bmin}, of the stiffness matrix $\underset{\sim}{B}$. We have no desire to calculate the entire spectrum, for that would be computationally prohibitive for the following reasons. First, the value of N may be quite large. Secondly, the stiffness matrix $\underset{\sim}{B}$ is nonlinear (in the displacement vector $\underset{\sim}{x}$) and can be expected to change dramatically at times during the dynamic relaxation iterative procedure. Its eigenvalue spectrum will shift accordingly. We, therefore, settle for the prospect of calculating only the *minimum* and *maximum* eigenvalues in a reasonably expeditious manner.

Referring to Figure 7, the best value of λ_{DR} occurs when $h^2\lambda_B/\rho = 2$. From this observation, it is reasoned that optimal values for the convergence parameter h^2/ρ be calculated by the formula:

$$\left(\frac{h^2}{\rho}\right)_{opt} = \frac{4}{\lambda_{Bmax} + \lambda_{Bmin}}$$

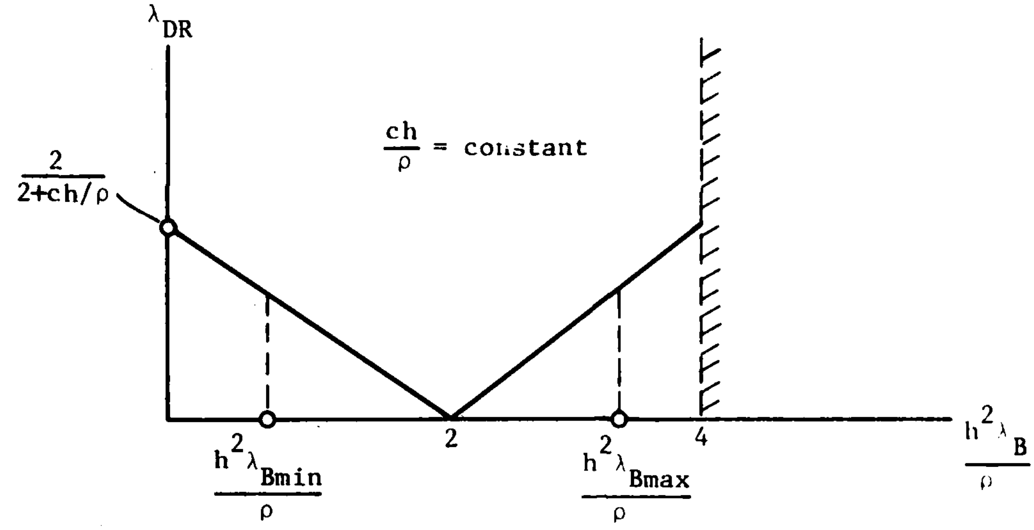


Figure 7. Behavior of decay rate and eigenvalues.

That is, this convergence parameter is calculated so that the average of the two abscissa values that are based on the minimum and maximum eigenvalues of the stiffness matrix, shall equal two. Accordingly, this formula gives values for the convergence parameter that should correspond to a minimum value for the decay rate λ_{DR}, the condition that we are trying to promote.

Also, from the second case we recall that:

$$\left(\frac{ch}{\rho}\right)^2 = \frac{\lambda_B h^2}{\rho}\left(4 - \frac{\lambda_B h^2}{\rho}\right)$$

Substituting the optimal expressions for the parameter h^2/ρ in this equation we are led to:

$$\frac{ch}{\rho} = \frac{4}{\lambda_{Bmax} + \lambda_{Bmin}} \sqrt{\lambda_B(\lambda_{Bmax} + \lambda_{Bmin} - \lambda_B)}$$

Whether or not we let $\lambda_B = \lambda_{Bmax}$ or $\lambda_B = \lambda_{Bmin}$ in this expression, we will arrive at the very same result for the optimal expression for the parameter ch/ρ,

$$\left(\frac{ch}{\rho}\right)_{opt} = \frac{4}{\lambda_{Bmax} + \lambda_{Bmin}} \sqrt{\lambda_{Bmax} \lambda_{Bmin}}$$

The two formulas for $(h^2/\rho)_{opt}$ and $(ch/\rho)_{opt}$ govern the values for the artificial mass and damping parameters ρ and c, respectively, for the dynamic relaxation procedure. They reflect the state of the stiffness matrix $\underset{\sim}{B}$ through their use of the maximum and minimum eigenvalues of that matrix. Further, their derivation is based on a systematic analysis of stability and convergence of the dynamic relaxation procedure. When these two optimal formulas are used, the range of the decay rate will be:

$$0 < \lambda_{DR} < 1$$

which ensures stability of the iteration process. Also, an optimum convergence rate is encouraged since smaller values of λ_{DR} (within this range) are promoted by using these formulas.

Estimating the Maximum and Minimum Eigenvalues

The calculation methods for the maximum and minimum eigenvalues of the stiffness matrix (scaled) $\underset{\sim}{B}$ are described here. Because these values are likely to vary during the solution process and their calculation repeated, and because knowing eigenvalues exactly is not required to maintain stability of the process, we may use approximate calculation methods.

To ensure stability we need only ensure that the inequality

$$\lambda_{DR} > \lambda_{Bmax}$$

is maintained during iteration with the dynamic relaxation method. Thus, if we are able to calculate an upper bound for the maximum eigenvalue, then we are assured that when its product with the term h^2/ρ is taken to be less than four, the product of λ_{Bmax} with h^2/ρ will also be less than four.

We have available the Gerschgorin theorem to calculate an upper bound of the maximum eigenvalue of a square matrix. Applying it, we can write:

$$|\lambda_{Bmax}| < \max_{i} \sum_{j=1}^{N} |B_{ij}|$$

where the B_{ij} are the entries in the scaled stiffness matrix $\underset{\sim}{B}$. This states that the maximum over all rows of $\underset{\sim}{B}$ of the sum of the entries (their absolute values) in each row is an upper bound of λ_{Bmax}. The implementation of this simple inequality must involve an algorithm that recognizes the matrix $\underset{\sim}{B}$ is never formed explicitly in the dynamic relaxation method.

A strength of the dynamic relaxation procedure is that the global stiffness matrix $\underset{\sim}{B}$ need never be formed explicitly, because calculating the residual $\underset{\sim}{r}$ involves only carrying out the product of the stiffness matrix and the displacement vector. This can be done at the local level, and therefore having to assemble the stiffness matrix is avoidable in the dynamic relaxation method. To do so is desirable and results in a storage savings advantage over competing direct solution methods.

Calculation of the minimum eigenvalue is less critical than the calculation of the maximum eigenvalue for it cannot directly affect the stability of the iteration process. Exact methods for its calculation are not required either, and neither are bounds necessary for its estimation. The minimum eigenvalue must only lie in the range

$$0 < \lambda_{Bmin} < \lambda_{Bmax}$$

Its value, along with that for the maximum eigenvalue, does affect the rate of convergence of the solution. Numerical experiments have demonstrated that poor estimates can adversely affect convergence, though they cannot directly cause a solution to blow up. Good estimates of the minimum eigenvalue should therefore be sought.

Two formulas for approximating the minimum eigenvalue are presented. The first is recommended by Lynch (1968) and Papadrakakis (1981) and the second is new. Any admissible value for λ_{Bmin} may be used at the start of the dynamic relaxation solution process. It is then assumed that while the solution is in progress, the following quotient approximately describes the decay rate, λ_{DR}

$$\frac{||\underset{\sim}{x}^{n+1} - \underset{\sim}{x}^{n}||}{||\underset{\sim}{x}^{n} - \underset{\sim}{x}^{n-1}||} \approx \lambda_{DR}$$

Further, it is assumed that this quotient becomes constant within some user-defined tolerance, after a sufficient number of iterations have passed. When that condition occurs it is assumed that the dominant eigenvalue of the stiffness matrix should correspond to the minimum eigenvalue. The first formula comes from using the previously given quadratic equation in λ_{DR} and solving for λ_{Bmin}, i.e.,

$$\lambda_{Bmin} = \frac{-(\lambda_{DR}^2 - \lambda_{DR}\,\beta + \alpha)}{\lambda_{DR}\,\gamma}$$

The quotient given for λ_{DR}, is used in this formula to estimate λ_{Bmin}. Then λ_{Bmin} can be used to update the optimal convergence parameters $(h^2/\rho)_{opt}$ and $(ch/\rho)_{opt}$.

The second formula is derived based on the further observation that it is the case of equal roots (in the quadratic equation solution) from which we have derived the formulas for the optimal convergence parameters. Therefore it is more consistent to use the formula which relates λ_{DR} and λ_{Bmin} from that case. Thus, the second formula for calculating the minimum eigenvalue is:

$$\lambda_{Bmin} = \frac{\rho}{2}\left[2 - \left(2 + \frac{ch}{\rho}\right)\lambda_{DR}\right]$$

Once again the solution vector quotient is used for determining λ_{DR} in this formula.

Unfortunately, in either formula, nonpositive values for λ_{Bmin} are possible and experience has shown that they will occur. A vigilance must be maintained for this condition and corrective action taken immediately. From the above formula, this condition seems to be avoided so long as:

$$\lambda_{DR} \approx \frac{||\underset{\sim}{x}^{n+1} - \underset{\sim}{x}^{n}||}{||\underset{\sim}{x}^{n} - \underset{\sim}{x}^{n-1}||} < \frac{2}{2 + ch/\rho}$$

A procedural description of the automated dynamic relaxation solution method is given by Algorithm 3.

Algorithm 3. Automated Dynamic Relaxation

Given $h = 1$ and $\underset{\sim}{F}_{ext}$, $\frac{h}{2\rho}$, $\frac{ch}{\rho}$, n_{max}

0. Initialize $n \leftarrow 0$, $\underset{\sim}{x} \leftarrow \underset{\sim}{0}$, $\underset{\sim}{F}_{int} \leftarrow \underset{\sim}{0}$

1. $\underset{\sim}{r}^0 \leftarrow \underset{\sim}{F}^0_{int} - \underset{\sim}{F}_{ext}$

2. $\underset{\sim}{x}^1 \leftarrow \underset{\sim}{x}^0 - \frac{h}{2\rho}\, \underset{\sim}{r}^0$ and $\underset{\sim}{x}^{-1} \leftarrow \underset{\sim}{x}^1$

3. $\underset{\sim}{x}^{n+1} \leftarrow \left(\frac{4}{2+ch/\rho}\right) \underset{\sim}{x}^n - \left(\frac{2-ch/\rho}{2+ch/\rho}\right) \underset{\sim}{x}^{n-1} - \left(\frac{2h^2/\rho}{2+ch/\rho}\right) \underset{\sim}{r}^n$

4. If convergence O.K. then output $\underset{\sim}{x}^{n+1}$ and stop.

5. If $n = n_{max}$ then output error message and stop.

6. $n \leftarrow n + 1$

7. $\underset{\sim}{x}^n \leftarrow \underset{\sim}{x}^{n-1}$ and $\underset{\sim}{x}^{n-1} \leftarrow \underset{\sim}{x}^{n-2}$

8. Calculate $\underset{\sim}{F}^n_{int}$

9. $\underset{\sim}{r}^n \leftarrow \underset{\sim}{F}^n_{int} - \underset{\sim}{F}_{ext}$

10. If convergence rate O.K. then go to step 3.

11. Estimate upper bound of maximum eigenvalue and estimate minimum eigenvalue.

12. Update parameters ch/ρ and h^2/ρ.

13. Go to step 3.

RESULTS FROM NUMERICAL EXPERIMENTS

The three solution algorithms described in this report were tested in the framework of small, two-dimensional, nonlinear finite element computer programs for the nonlinear static analysis of planar cables. These programs were developed and written specifically for that purpose. There was an interest in keeping the programs as similar as possible. However, the ADR algorithm's implementation was more lengthy. The implementations were carried out in UCSD Pascal on a Stride Micro 440 using the p-System operating system.

The results for the modified Newton algorithm were so unsatisfactory over the range of test problems that they are omitted. This algorithm does have attributes but robustness relative to highly nonlinear problems is not one of them.

Robustness Behavior

The set of test problems was primarily designed to test an algorithm's ability to seek the correct static solution while starting from an initial cable configuration that is arbitrarily prescribed. Thus, the robustness of the algorithm is addressed in this set of test problems by deliberately specifying onerous initial configurations. In all cases, the unstrained cable length, which in practical engineering problems is generally a known quantity, is treated as prescribed input data. This information along with prescribed external loads and/or cable span lengths are used to compute the static equilibrium configuration with its corresponding state of prestress. These are all examples of Phase I tensioned structure problems.

There were 14 cable test problems, which were grouped in the following four subclasses.

1. Fixed-Span Suspended Cable
2. Cable Snap-Through
3. Mooring Cable
4. Varying-Span Suspended Cable

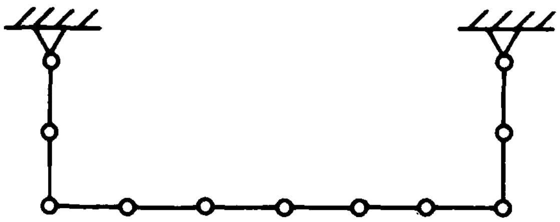
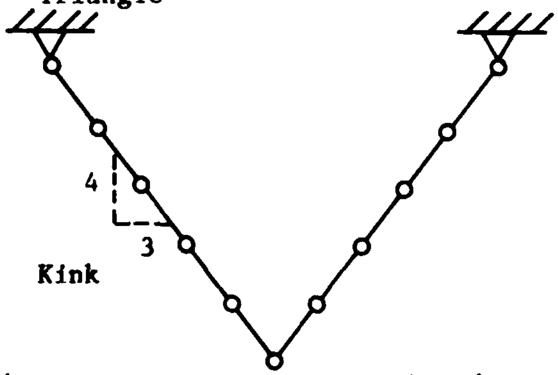
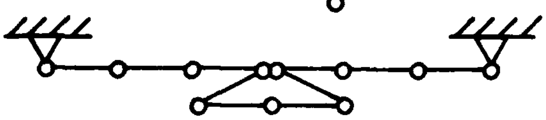
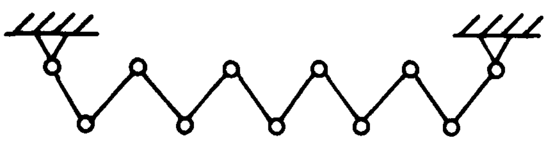
These problem classes and their corresponding solutions, and the performance of the algorithms are each described separately.

Fixed-Span Suspended Cable Problem

This test problem consists of a cable suspended between two supports and acted on by lateral, unit forces at each node. The rigidity of the cables, EA, is 1000. The unstrained length of the cable is 100 units, and the span is 60 units. The cable length is uniformly divided into 10 elements. Table 2 compares the convergence of the full Newton and ADR algorithms for four initial configuration cases. The results indicate that convergence to the correct solution was achieved successfully by both algorithms for all cases except the third, the kinked initial configuration case. This case consists of an initial configuration where all the elements lie on a straight line across the span, and the two inner most elements are exactly colinear (though they are shown parallel) with and overlap adjacent elements.

The expected equilibrium solution for this problem is presented in Figure 8. Also shown is the unexpected equilibrium position found by the full Newton algorithm for certain prescribed initial prestress forces. In this solution the two overlapped elements are sustaining compressive forces. Perhaps the full Newton algorithm would have found the expected solution if there had been provision for tension-only element behavior. However, the ADR algorithm required no such provision, and further, is independent of initial prestress force. It, thereby, exhibits greater robustness.

Table 2. Numerical Results of Fixed-Span Suspended Cable

Case No.	Initial Configuration	Algorithms	
		Full Newton	ADR
1	Rectangle 	Converged	Converged
2	Triangle 	Converged	Converged
3	Kink  (all elements are colinear)	Can converge to unexpected solution	Converged
4	Saw tooth 	Converged	Converged

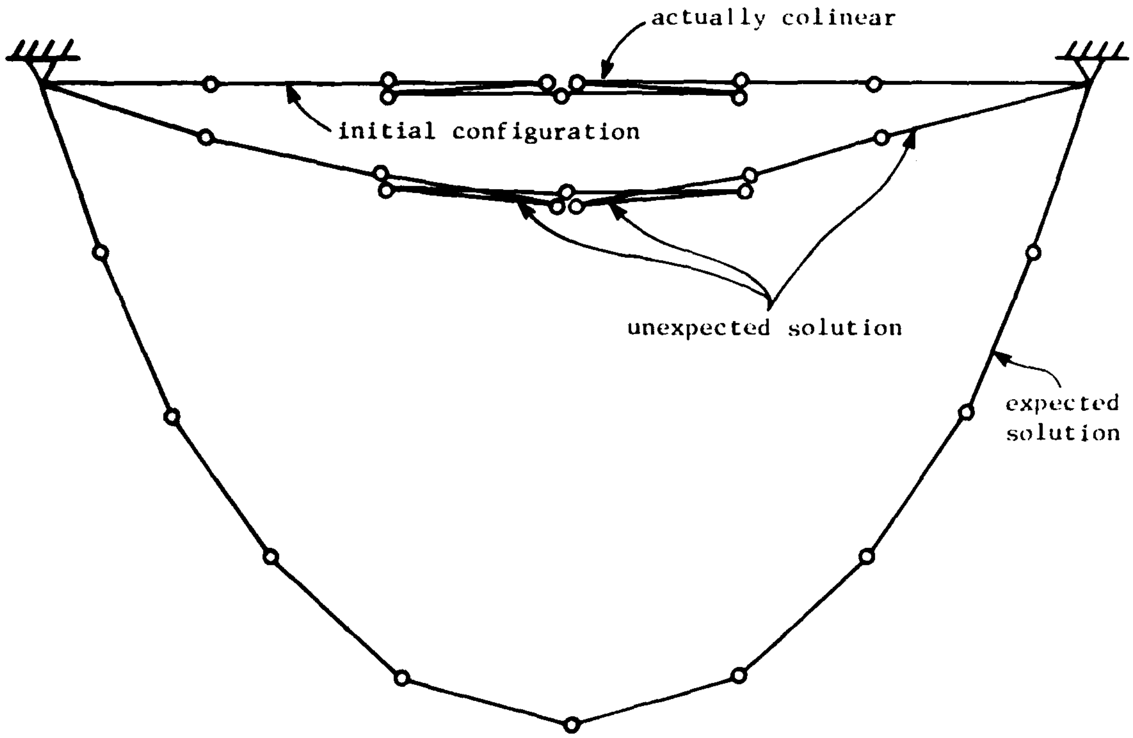
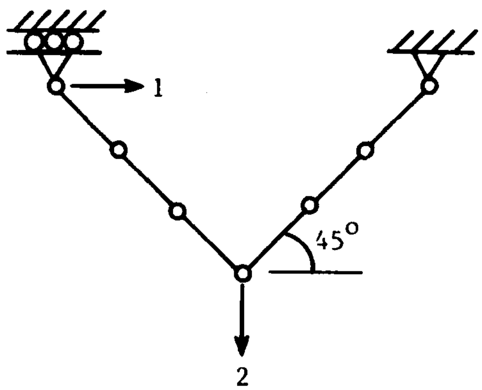
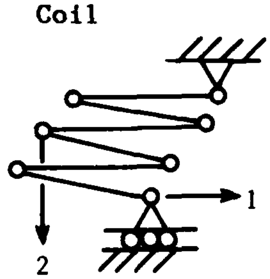


Figure 8. Expected and unexpected equilibrium configuration for kinked cable.

Cable Snap-Through Problem

This test problem consists of a six-element suspended cable with an unstrained length of 60 units. The cable is considered weightless with an EA value of 1000, and subjected to concentrated horizontal and vertical forces as shown in Table 3. The end support that is subjected to the horizontal force is free to move in the horizontal direction beginning from either of two prescribed initial configurations until a static equilibrium configuration is reached. In the second case, the six-cable elements are initially coiled on top of one another such that their nodes possess exactly the same initial coordinate values. As the solution algorithm seeks equilibrium, the cable tends to uncoil and snap through from left to right.

Table 3. Numerical Results - Cable Snap Through

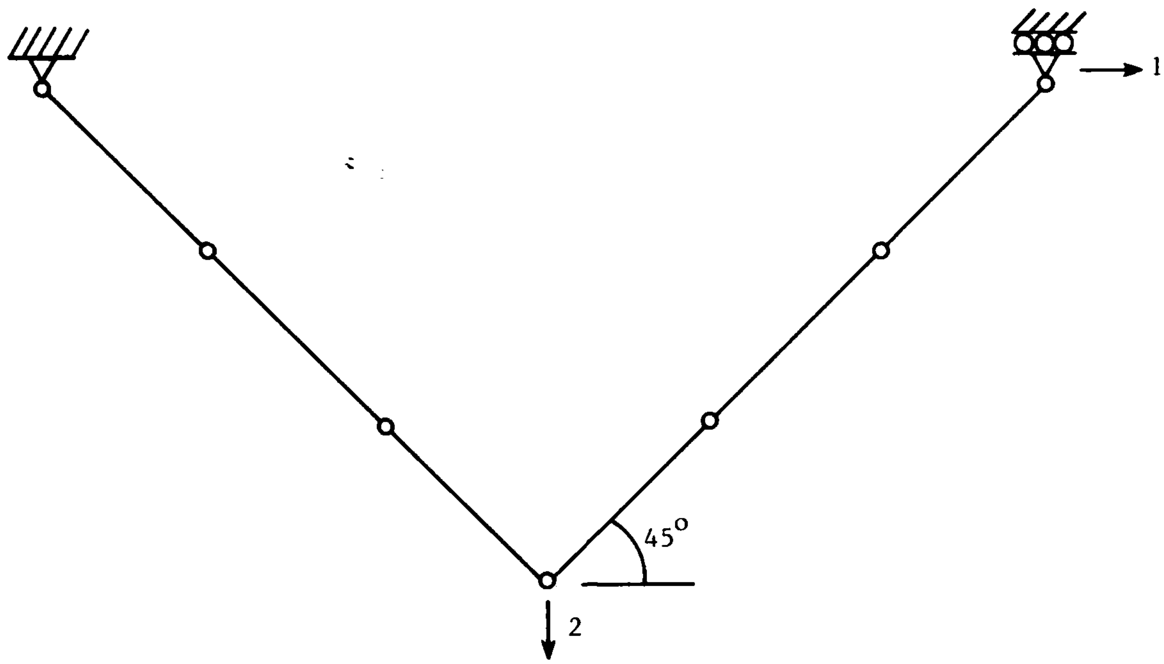
Case No.	Initial Configuration	Algorithms: Full Newton	Algorithms: ADR
1	Triangle 	Converged	Converged with poor accuracy
2	Coil  (all elements are colinear)	Can converge to unexpected solution	Converged

The ADR algorithm converges to the expected equilibrium solution, which is shown in Figure 9a, from either initial configuration. However, in the first case, the final calculated span was short by 9 percent. This is an anomalous result for in all other test problems no such inaccuracy occurred with the ADR algorithm. The accuracy of the full Newton algorithm was good in this case. In the second case, however, the full Newton algorithm did not always converge to the expected solution depending on the value of the initial prestress force prescribed. The ADR algorithm is independent of this quantity, and converged to the expected solution (accurately). Two, unexpected, alternative equilibrium states found by the full Newton algorithm are shown in Figures 9b and 9c. Once again these states include elements sustaining compression forces.

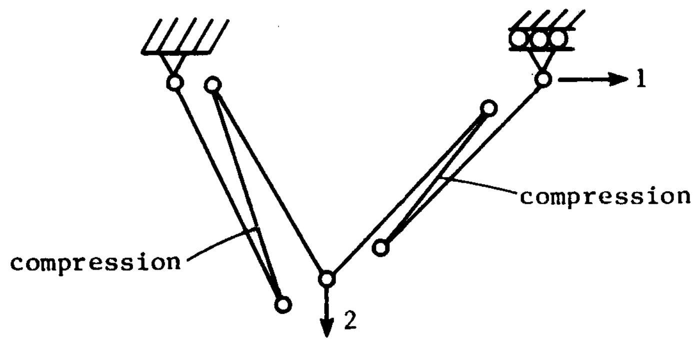
Mooring Cable Problem

This test problem simulates a highly idealized, single leg mooring cable, which is a common structure in ocean engineering. Five different initial configuration cases were designed as indicated in Table 4. Cases 1 through 4 all possess an inverted L-shaped initial configuration but may differ in the specification of current forces applied to the cable or in the cable elastic modulus. In these cases, the unstrained cable length is 900 feet and is uniformly subdivided into six elements. Case 5 possesses a vertical initial configuration and is referred to as a taut system. In this case, the unstrained cable length is 600 feet. In all cases the lower support is fixed (anchored), and the upper support is constrained to slide horizontally along the surface under the action of constant current forces and a 500-pound surface force.

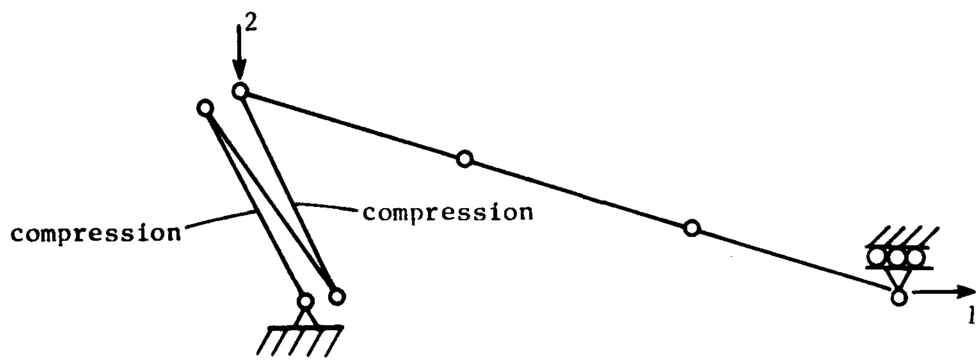
Table 4 indicates that for all five cases, both the full Newton and ADR algorithms converged to the expected equilibrium solutions. The expected solutions are shown in Figure 10. Figure 10a shows the equilibrium solutions computed for cases 1 through 4. The prescribed cable



(a) expected



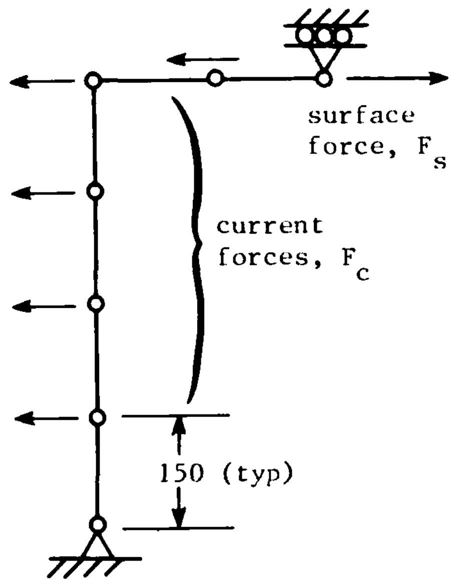
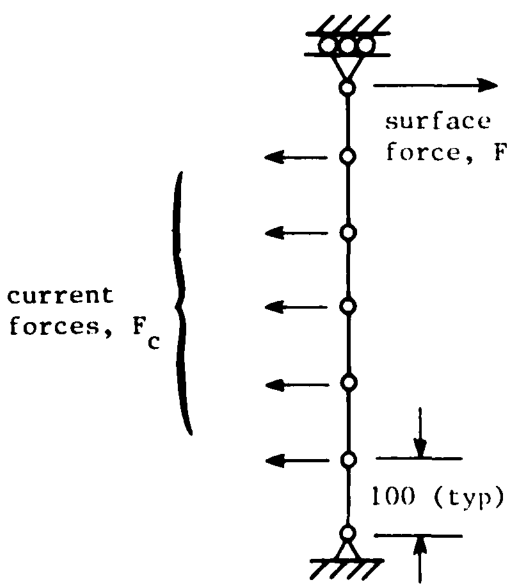
(b) unexpected

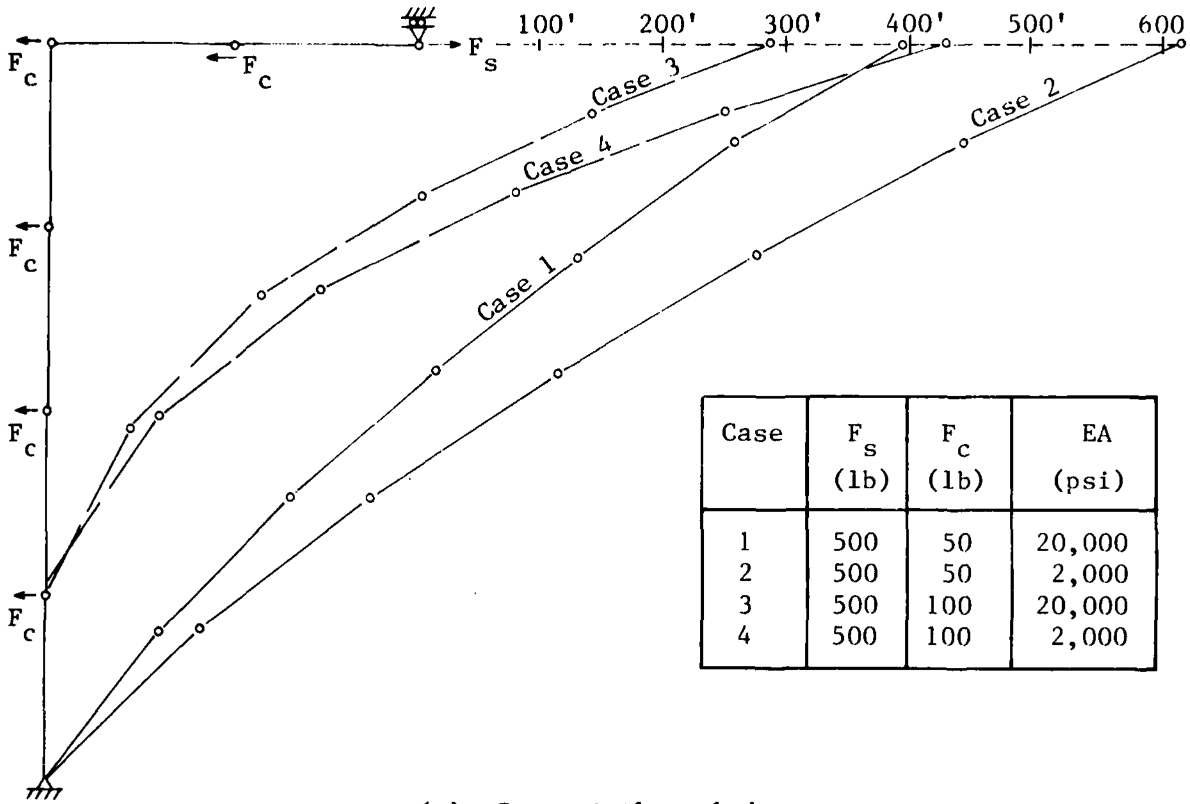


(c) unexpected

Figure 9. Expected and unexpected equilibrium configurations for coiled cable.

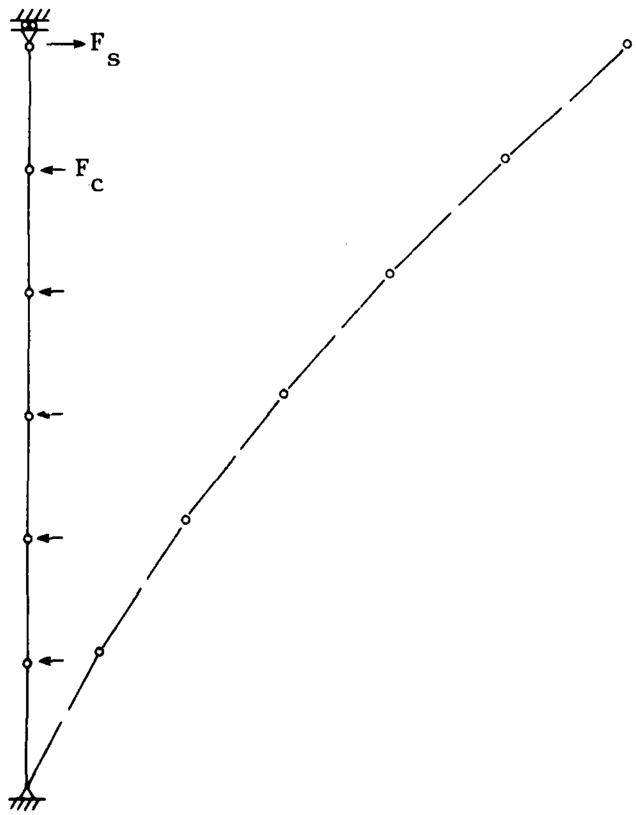
Table 4. Numerical Results - Mooring Cable

<table>
<tr><th rowspan="2">Case No.</th><th rowspan="2">Initial Configuration</th><th colspan="2">Algorithms</th></tr>
<tr><th>Full Newton</th><th>ADR</th></tr>
<tr><td>1-4</td><td>Inverted L-shape

</td><td>Converged</td><td>Converged</td></tr>
<tr><td>5</td><td>Taut

</td><td>Converged</td><td>Converged</td></tr>
</table>



Case	F_s (lb)	F_c (lb)	EA (psi)
1	500	50	20,000
2	500	50	2,000
3	500	100	20,000
4	500	100	2,000

(a) Cases 1 through 4.



F_s = 500 lb

F_c = 50 lb

EA = 2,000

(b) Case 5.

Figure 10. Equilibrium configurations for a mooring cable.

moduli are soft, so large strains*, as well as, large displacements are developed. The surface node point, for example, has moved more than 600 feet from its initial position in Case 2. Figure 10b is the computed solution for the taut cable, Case 5.

It was clear from these results that the more taut the cable system, the easier it was for the algorithms to converge upon the equilibrium solution. In this respect, these were well behaved problems and consequently the solution algorithms were not really challenged.

Varying-Span Suspended Cable Problem

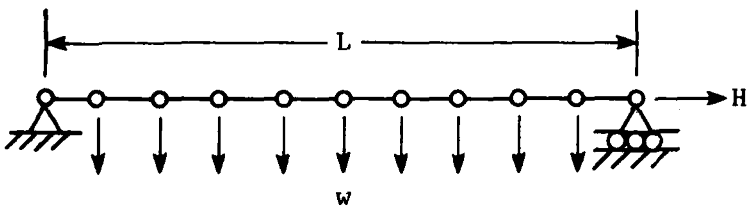
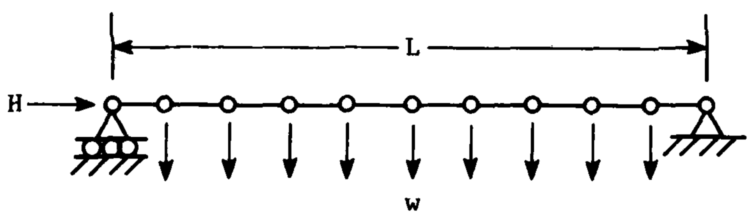
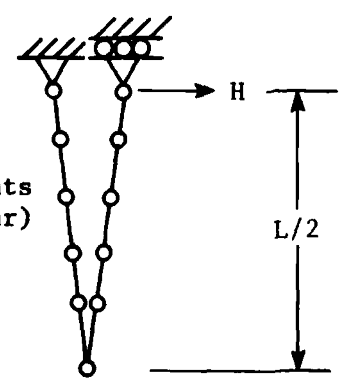
This test problem consists of a horizontally suspended cable with one of two supports free to slide horizontally. The cable is acted upon by uniform lateral forces and a concentrated, horizontal force both of which cause one support to slide until the system reaches equilibrium. The unstrained cable length is 200 feet, and it is uniformly subdivided into 10 elements. This test problem was used by Webster (1979) in a similar study of solution algorithms for ocean cable systems.

The three initial configuration cases studied are shown in Table 5. Also given in the table are the problem parameters including the cable rigidity, EA. In the first two cases, the cable is initially aligned exactly horizontally along the span. Case 1 is labeled taut because its initial configuration is such that the cable will remain in tension throughout the solution process. Case 2 is labeled slack because during the solution the cable must pass from a compression state to a state of full tension. In Case 3, a completely sagged cable is represented wherein all elements initially lie along a common vertical line. However, the cable should remain in tension throughout the solution process.

The ADR algorithm converged to the expected equilibrium solution in all three cases. The expected solution is shown in Figure 11. However, in Case 2, the full Newton algorithm did not always converge to this

*The computer programs used do not possess a large strain formulation in which the cross sectional area of the cable becomes variable. The cross-sections remain constant in the present programs, and they are only applicable to small strain problems.

Table 5. Numerical Results - Varying-Span Suspended Cable

<table>
<tr><th rowspan="2">Case No.</th><th rowspan="2">Initial Configuration</th><th colspan="2">Algorithms</th></tr>
<tr><th>Full Newton</th><th>ADR</th></tr>
<tr><td>1</td><td>Taut
</td><td>Converged</td><td>Converged</td></tr>
<tr><td>2</td><td>Slack
</td><td>Can converge to unexpected solution</td><td>Converged</td></tr>
<tr><td>3</td><td>Sag
(all elements are colinear)
</td><td>Converged</td><td>Converged</td></tr>
</table>

L = 200 ft
H = 5.77 lb
w = 0.1 lb/ft
EA = 1 x 10^3 lb

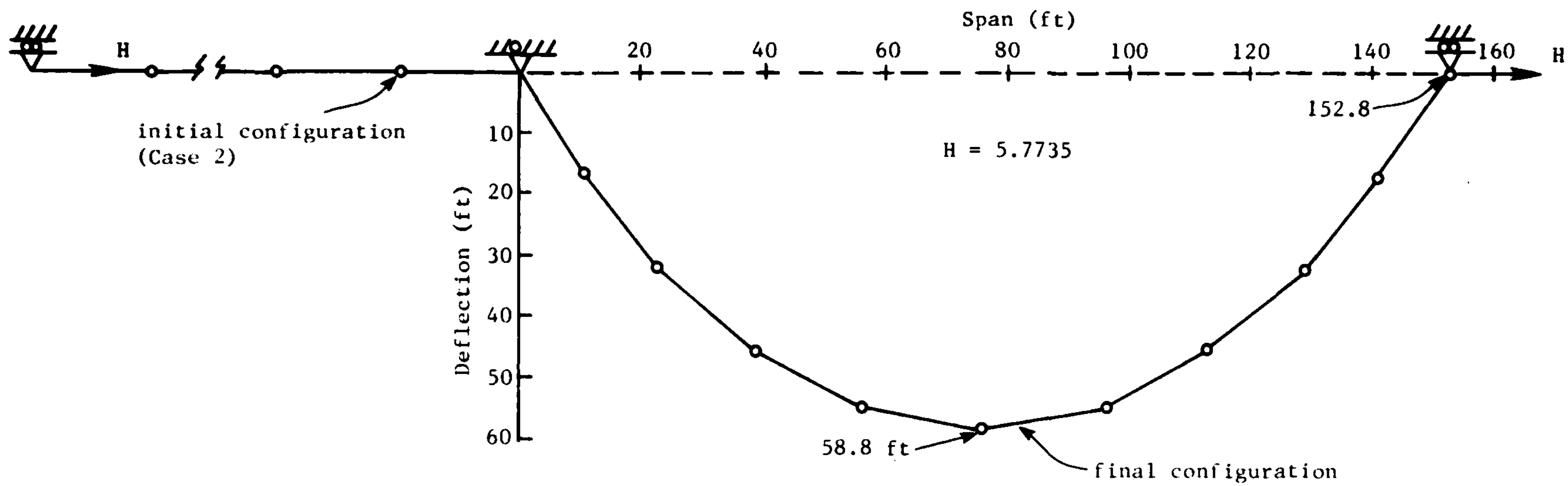


Figure 11. Expected equilibrium configuration for varying span problem.

solution. Its behavior, in this case, was like that already discussed for the cable snap-through problem. Whether the expected solution was found or not depended on the value of the prescribed prestress force. Webster (1979) showed that the full Newton algorithm failed for this test case. Here, it was learned that it may or may not fail. In general, however, these results are in agreement with those of Webster's concerning the unrealibility of the full Newton algorithm for this class of problems.

The ADR algorithm is more robust than the full Newton algorithm in seeking the expected equilibrium solution in the varying-span suspended cable problem. Once again, this behavior is correlated with whether or not a slack cable configuration arises in the solution process. Also the results from Cases 1 and 3 in this test problem show that the ability of either algorithm to converge is not dependent on the degree of cable sag present in the initial configuration. Successful convergence was achieved by either algorithm for no initial cable sag in Case 1, and for maximum cable sag in Case 3.

Recall that cable slackness is a mechanical condition, and that cable sag is a geometrical condition. This test problem demonstrates that it does matter whether or not cable slackness is present, but it does not matter to what degree cable sag is present.

Performance Behavior

Though the main goal in this study centers on the robustness behavior of the algorithms, their performance behavior, i.e., their speed of convergence, also deserves some discussion. The full Newton algorithm converges much faster than the ADR algorithm according to the results of this study.

Table 6 contains the number of iterations that was required by the full Newton and the ADR algorithms to converge to the expected solution in the test problems. The time required for one iteration through each algorithm was comparable, with the ADR algorithm taking only a little longer. Clearly the full Newton algorithm was one to three orders of magnitude faster for problems where it was also able to converge to the

Table 6. Convergence Performance[a]

Problem No.	Case No.	Initial Configuration	Full Newton Iteration[b]	ADR Iteration
1	1	Rectangle	7	719
2	2	Triangle	8	608
3	3	Kink	--[c]	1121
4	4	Sawtooth	14	862
5	1	Triangle	6	9736[d]
6	2	Coil	69	8089
7	1	Stiff	9	387
8	2	Softest	7	80
9	3	Stiffest	15	1209
10	4	Soft	9	125
11	5	Taut	24	101
12	1	Taut	5	2290
13	2	Slack	--[c]	5320
14	3	Sag	3	3710

[a]Convergence criterion was 10^{-4} on residual norm.

[b]Two load steps were employed.

[c]Convergence to unexpected solution.

[d]Converged solution was not accurate.

expected solution. This comparison might be distorted somewhat by a less than optimal implementation of the ADR algorithm, but it's doubtful that improvements on the implementation would make up the difference in performance shown here. A more reasonable explanation of the difference probably lies in the fact that the test problems used here were very small problems.

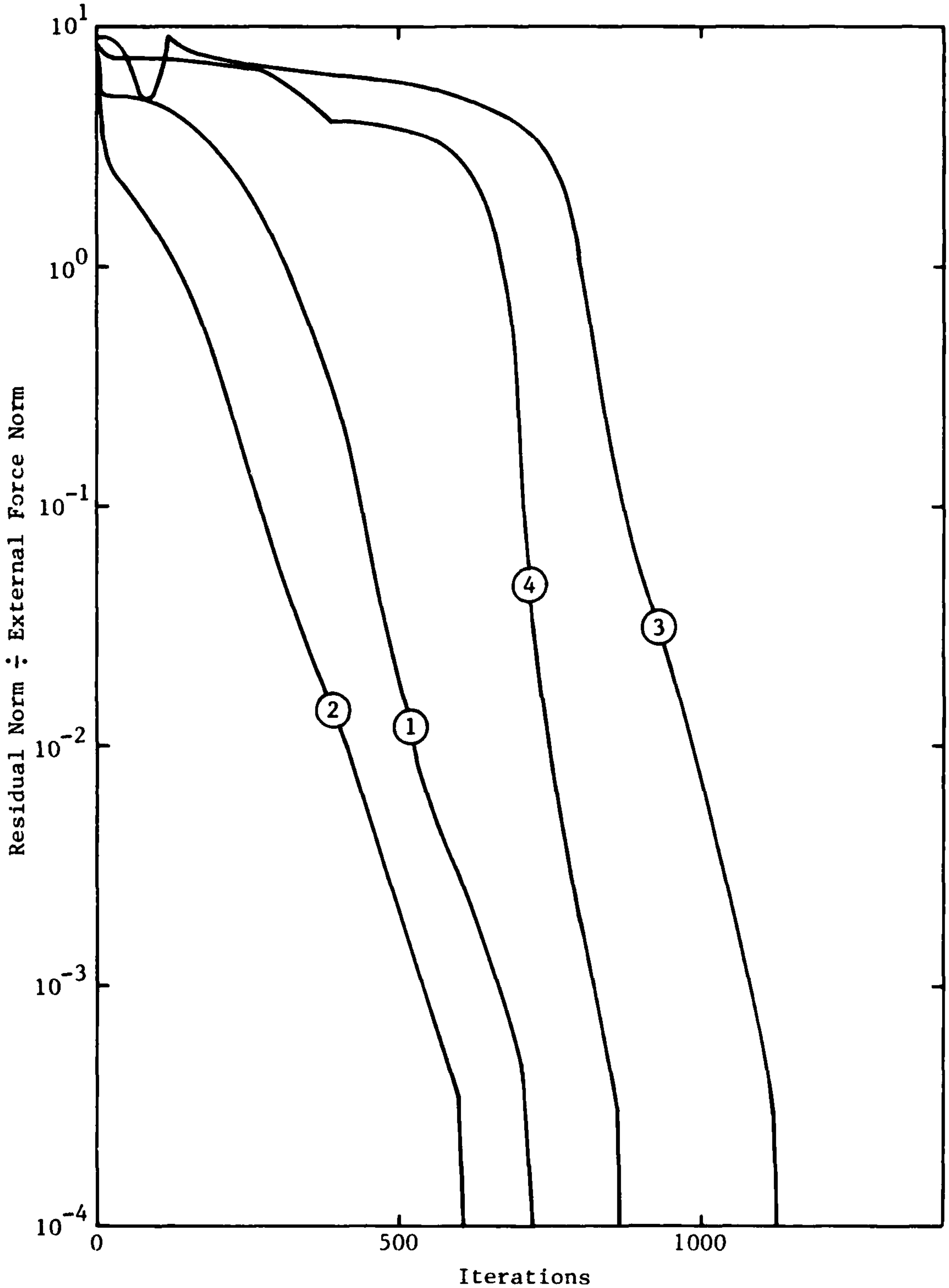
In a similar study using larger, three-dimensional, nonlinear cable problems, Papadrakakis (1981) reports a more favorable comparison of convergence speeds between the two algorithms studied, although even in these results, the full Newton algorithm is faster. As the system matrices become larger, the necessity for their triangularization and factorization becomes more of a burden to the full Newton algorithm, and rapidly slows it down. The same rate of decrease in convergence speed is not experienced by the ADR algorithm since it does not require these expensive matrix operations.

For very large, highly nonlinear problems the speed of convergence for the ADR algorithm should be comparable to the full Newton algorithm. However, most cable structures are generally considered to be no more than moderately large in comparison to other types of three-dimensional systems. Seldom does a cable structure's finite element idealization possess more than 2,000 degrees of freedom, while this is otherwise a common occurrence in finite element analysis.

A possible mitigating influence on the speed of convergence of the full Newton algorithm is that its natural quadratic convergence rate may not be realizable for ill-conditioned systems. Even though Papadrakakis' study considered larger cable test problems, they must also be regarded as well conditioned, taut cable problems. He was not emphasizing the robustness issue as has been done in the present study. Thus, the full Newton algorithm would definitely be expected to perform at its best when compared to the ADR algorithm. It is noted that in the present study, whenever a slack cable condition was involved the speed of both algorithms would deteriorate. However, this adverse effect seems more pronounced for the full Newton algorithm. The results of Problems 5 and 6 presented in Table 6 for the full Newton algorithm, gives evidence of this deterioration. The algorithm was successful in this case, but the number of iterations was inordinately large for Problem 6.

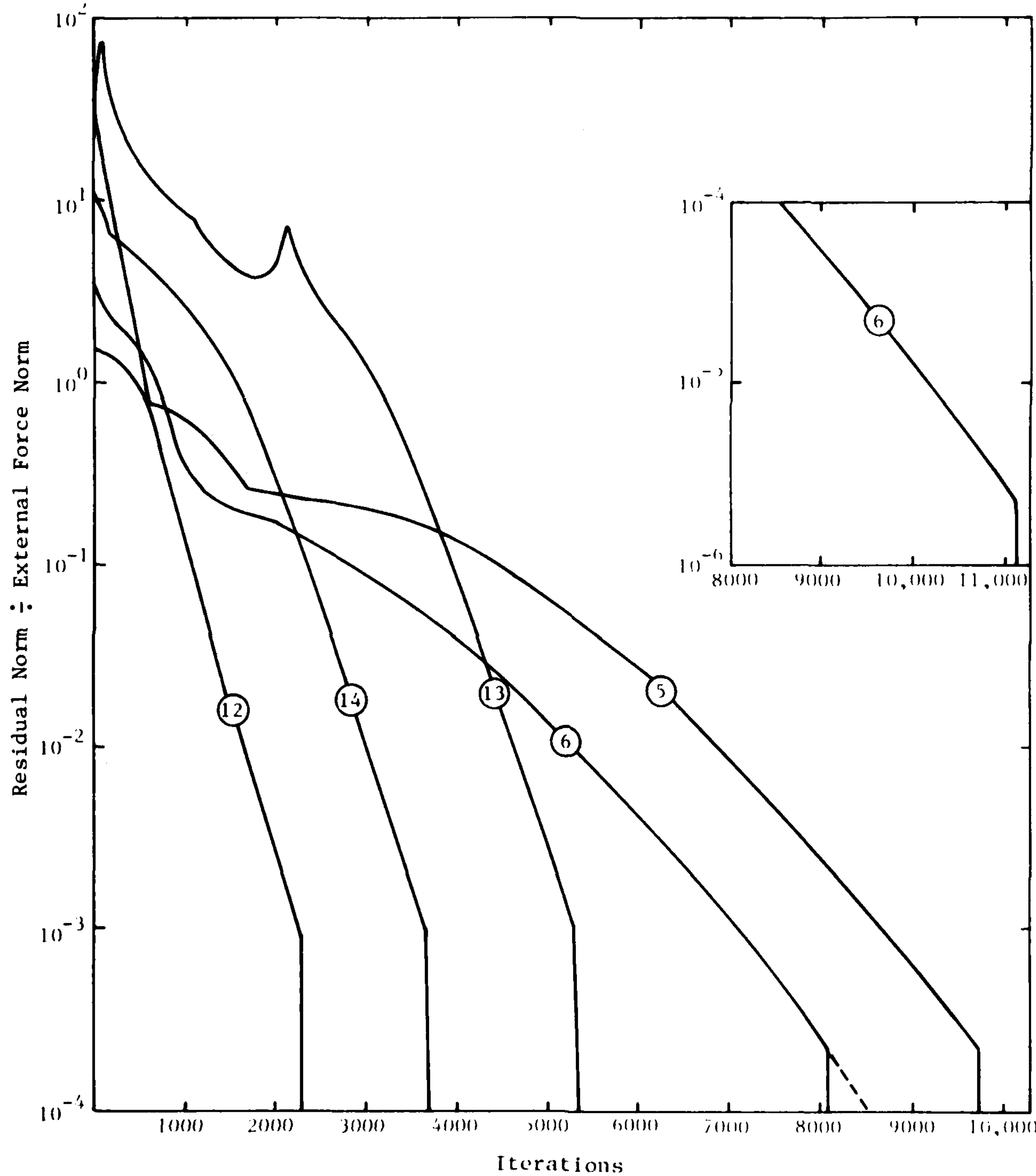
The rate of decay of the norm (Euclidian) of the residual force vector for the ADR algorithm is reflected in Figure 12 for each of the 14 test problems. While the ADR algorithm is clearly slow, its tenacity is also clearly demonstrated in these graphs, particularly for the ill-conditioned Problems 3, 5, 6, and 13.

In general, however, it should be expected that the full Newton algorithm will out perform the ADR algorithm in terms of speed of convergence over the range of problems involving well-behaved, engineering cable structures. It is well known to possess a quadratic convergence rate. When the test problems are small, as they were in this study, it is extremely fast in comparison with the ADR algorithm.



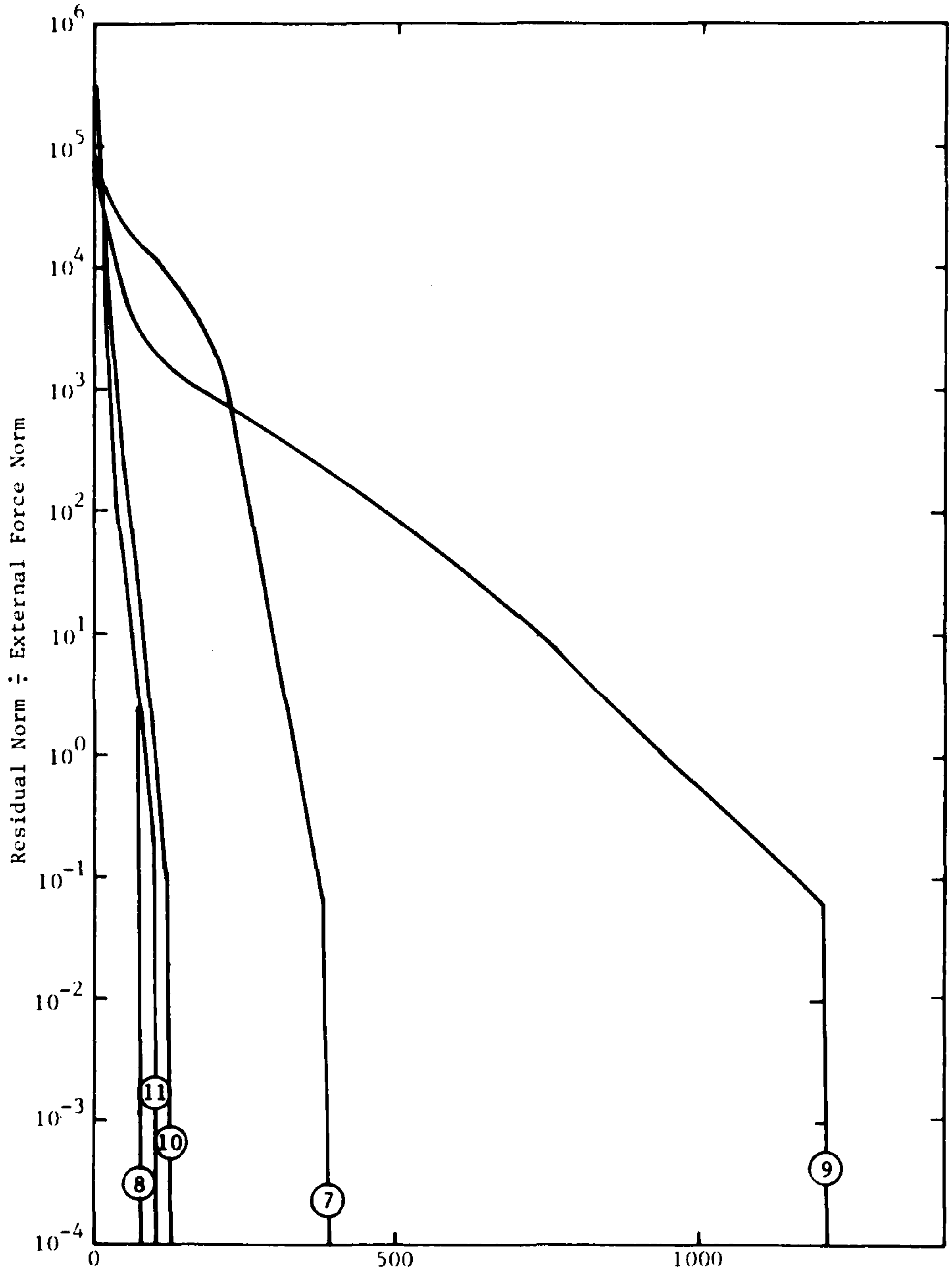
(a) Problems 1, 2, 3 and 4.

Figure 12. Convergence performance of ADR.



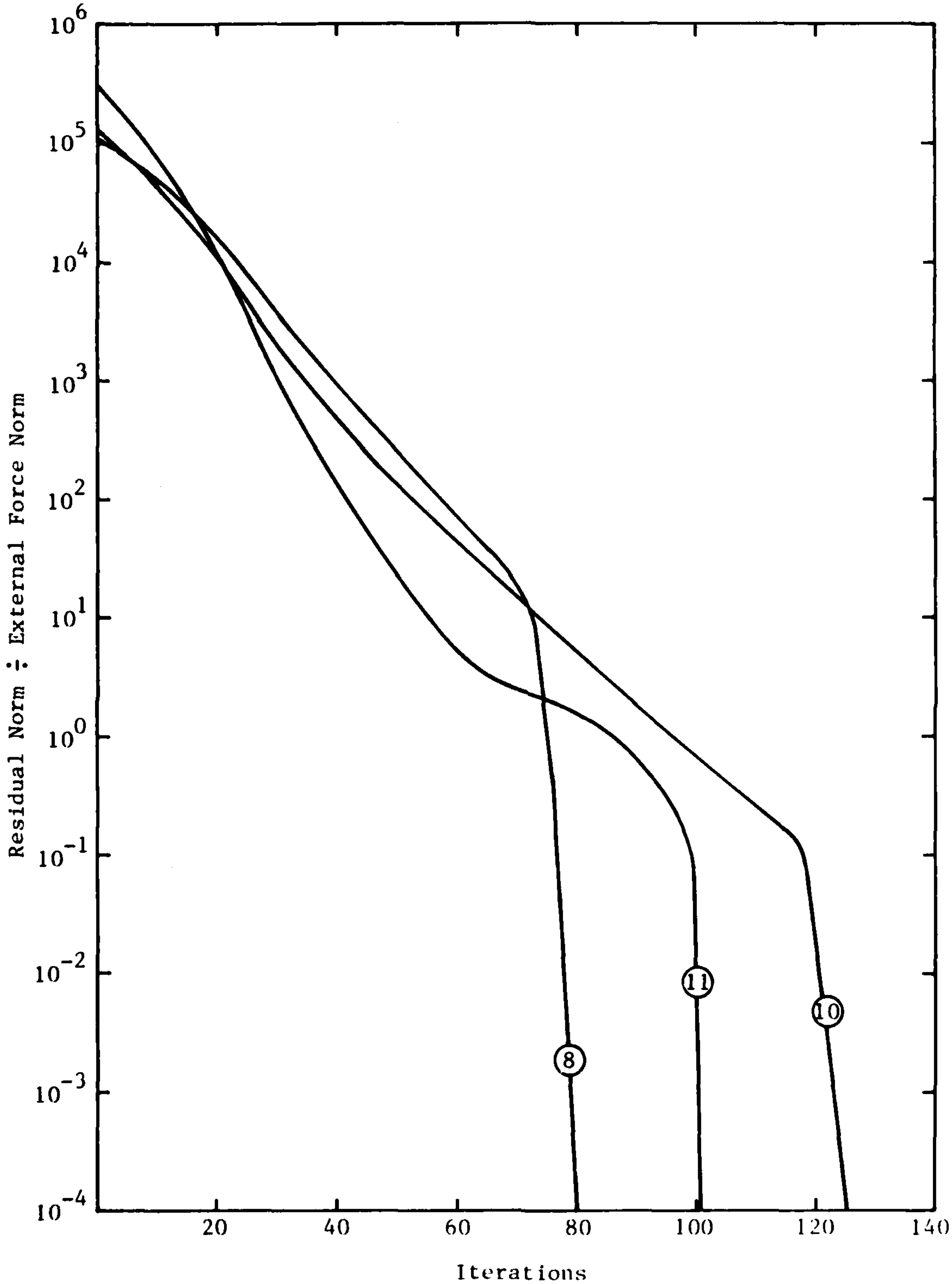
(b) Problems 5, 6, 12, 13 and 14.

Figure 12. Continued.



(c) **Problems 7, 8, 9, 10 and 11.**

Figure 12. Continued.



(d) Close up of Problems 8, 10 and 11.

Figure 12. Continued.

SUMMARY AND CONCLUSIONS

Solution algorithms for nonlinear static cable problems were theoretically studied, developed, and implemented by writing separate but similar finite element computer programs to test and compare their robustness characteristics. This class of problem is referred to as a Phase I problem in the context of a two-phase problem organization of the general solution of tensioned structures. Solutions to the Phase I problems are widely regarded as a stumbling block in engineering design and analysis of tensioned structures, i.e., ocean cable structures and land-based fabric and cable structures.

Three iterative solution algorithms were studied: the modified Newton algorithm, the full Newton algorithm, and an automated dynamic relaxation (ADR) algorithm. The first two have been used in nonlinear finite element programs for many years and may be regarded as status quo solution algorithms. The third algorithm is a promising solution algorithm for problems involving highly kinematically nonlinear structural behavior.

The ADR algorithm possesses some attractive theoretical features relative to ill-conditioned systems. These features provide constant monitoring of the condition of the structure stiffness matrix with a corresponding ability to control the stability of the solution process automatically in a logical and systematic way. Typically, in Phase I problems, the stiffness matrix exhibits pathological behavior when slack conditions in the structure arise during the solution process. The status quo methods mentioned do not monitor and control the structure stiffness matrix.

The implementation of the ADR algorithm can be lengthy depending on the type and degree of monitor and control operations desired. This makes the ADR algorithm somewhat subjective and its implementation somewhat ad hoc. These characteristics would seem to diminish the chances for a robust solution procedure but they do not. Once these operations are in place, the algorithm is automatic and robust.

The status quo methods, whose algorithms seem more straight forward and not so subjective (this is less true of the modified Newton algorithm), have a feature that detracts from robustness. The Newton-based methods require that an arbitrary level of member prestress force be prescribed (along with the initial guessed configuration) to avoid a singularity condition in the cable structure's stiffness matrix at the beginning of the solution process. This study has shown that the robustness of these methods is adversely affected by this requirement for convergence to the expected equilibrium solution sometimes depended on the value of prescribed prestress. That is, any input data requirement that affects convergence in this way, runs contrary to the goal of a foolproof solution algorithm for Phase I problems.

A set of 14, small cable test problems (representative of Phase I problems) were designed to evaluate the robustness of the three algorithms studied. In these problems, the initial configurations were purposely designed to be onerous to test the ability of the algorithms to seek an expected static equilibrium configuration.

The numerical experiments revealed that the modified Newton algorithm was not competitive with the other two algorithms studied.

The ADR algorithm proved to be more robust than the full Newton algorithm. It converged to the expected static equilibrium configuration in every test problem. In one anomalous case, the accuracy was poor. Conversely, the full Newton method sometimes failed to converge to the expected solution in three cases. These cases involved conditions where the solution process was required to traverse a state of compression to a final state of tension, encountering slack conditions along the way. Convergence to unexpected and meaningless alternative equilibrium states occurred with the full Newton algorithm.

In those problems where both the ADR and full Newton algorithm successfully converged to the expected equilibrium configuration, the ADR algorithm was slower. So a trade-off between robustness and convergence speed will be expected when considering application of these two algorithms.

The results of the numerical experiments conducted in this study suggest that further consideration be given to both the ADR algorithm and the full Newton algorithm in future software development in support of ocean cable structures and land-based tensioned fabric and cable structures.

RECOMMENDATIONS

It is recommended that further development of the ADR procedure be pursued for future application in special purpose structural analysis software for tensioned structures.

Numerical experiments should be conducted with the SEADYN computer program using the test problems designed in this study. This program's performance can then be compared with the ADR algorithm.

Proceeded by some additional research and development, more substantial implementations of the ADR algorithm and the full Newton algorithm could be accomplished. This would allow further, more meaningful comparison of these solution algorithms with that in SEADYN. Larger, more physically meaningful, ocean cable problems should be designed and used for this study.

This additional research and development study should establish sound, robust solution algorithms around which future special purpose finite element software for ocean cable systems can be planned and developed.

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